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Use Patterns for Depletable and Recycleable Resources [1,2]

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1. INTRODUCTION

Much policy interest in the environment these days is focused on the attractiveness of recycling and reusing natural resources that are in fixed supply. Commoner [3] and others have elicited this interest by their discussion of the earth as an ecologically "closed" system. Much of this interest has been generated by the increasing economic and environmental costs of solid waste disposal, and the ever-sharpening awareness of those costs. The applause for municipal and civic recycling has surely stemmed primarily from their contributions to environmental preservation.[3]

The major long-term contribution of recycling may lie, however, in a related but different arena. It can increase the effective long-term supplies of the resources that are being reused. In this context there is no need for public action. When recycleable resources are able to secure the appropriate rentals, private enterprises can be counted upon to conduct the required reclamation processes. The evident expansion of such private efforts suggests that for many resources the appropriate rentals are presently upon us.

Market forces can be counted upon to bring about some recycling. Can they be relied upon to bring about the optimal amount? This paper addresses that question with the use of a simplified multi-stage model of resources in limited supply. The solid-waste problem, with its associated external diseconomies, is not considered.[4]

The goods that best illustrate our model under present economic conditions are iron, aluminium and a variety of heavy metals. For an explanation of the burgeoning interest in recycling these materials, we look to the supply and demand for these fixed-supply natural resources.

In the predecessor paper to this work [14] the analysis was directed to natural resources in fixed supply that are depletable in the strict sense. Oil, coal and natural gas well fit that model. For them, reuse or regeneration are ruled out. The major result of our strictly depletable resource model was that the resource's price, assuming a perfect competitive commodity and capital market, would reflect marginal social cost in terms of discounted consumer-plus-producer surplus.[5] The optimal (and under these conditions,

1 *First version received May* 1973; *final version accepted February* 1974 (*Eds.*).

2 This research was supported in part by the Analytic Methods Seminar, Kennedy School of Government, Harvard University. We are indebted to K. Arrow, G. M. Heal, P. Memishian, R. M. Solow, and to a referee for helpful comments, and to Thomas Shemo for computer assistance.

3 Sometimes the threat of governmental regulatory action or public outcry may spur a private organization to undertake a recycling effort whose primary objective is the provision of an environmental externality. Glass bottle recycling provides a good illustration; bottling concerns as well as civic authorities have undertaken these efforts.

See Benka [1] for a cost/benefit analysis that weighs the aesthetic gains against the economic costs of bottle recycling.

4 This quite different dynamic management problem is considered in Keeler, Spence and Zeckhauser [6].

5 We deal here only with the criterion of efficiency, assuming that redistribution is carried out elsewhere. However, if existing government transfer programmes are deficient, and if the consumption of a natural resource is a major determinant of real income, it would conceivably be appropriate to ration this resource as a redistributional measure. Prices and quantities could be held below market equilibrium levels through artificial restrictions on demand. (Future consumers would be among the primary, though unintended, beneficiaries of such a measure.)

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observed) price rises at the rate of interest. If extraction costs are positive, the difference between price and the appropriately defined marginal cost of extraction rises at this rate.[1]

The analysis of the previous paper [14], as well as its extension to recycling which forms the second major portion of the present paper, treats the interest rate as an exogenous variable. This treatment, quite obviously, depends on the assumption that the depletable (or recycleable) resource is of relatively minor consequence with respect to the total economy. Its minor role may derive from any of a number of factors. First, there may exist an ample supply of substitutes for the resource; its presence is a boon to the economy, but not a necessity. Second, the resource may be so plentiful relative to needs, as with sand for example, that for the foreseeable future it functions as if it were a reproducible good.[2]

In this paper, before turning to the analysis of recycling, we move beyond the framework where the rate of interest is determined independently of resource consumption. We examine a variety of models to see how the significant presence of depletable resources (and, by extension, recycleable resources) would affect market interest rates. We conclude from this analysis that our optimality and equilibrium results derived later on do indeed generalize to the case where interest rates are determined endogenously.

Then we turn to the analysis of recycling. The assumption of strict depletability can be relaxed to a variety of degrees. The polar opposite would be no depletion whatsoever. Diamonds are a fair example. Their value does not depreciate with use. Less extreme are situations where use does not exhaust a good, but does irrevocably deplete some fraction of it. Chemical compounds used as catalysts might lead such a physical existence.

The major recycling model deals with a third case. It is assumed that the commodity is in fixed supply, but that it can be recycled at a cost. Different analytic structures are shown to arise depending on the relationship of marginal extraction costs to the cost of recycling. This most general model can be applied to a wide range of mineral resources for which recycling and additional extraction are both possible. For many resources, the stage of interesting switchovers in economic behaviour may not have been reached. But as demand shifts outward, as costs of extraction increase, and as technological progress yields less costly recyling procedures, we can expect to observe an ever increasing array of materials that are recycled for their resource values.

In all of this analysis, capital markets and information are assumed to be perfect; storage costs to suppliers are assumed to be zero. It is shown in all cases that the socially optimal time stream of consumption is identical to the stream produced by perfectly competitive markets. The policy conclusion is, as in the case of absolutely depletable resources, that resource preservation considerations alone provide no justification for governmental intervention into the allocation processes of perfect free markets.

2. THE INTEREST RATE AS AN ENDOGENOUS VARIABLE

Our first task in this effort is to examine the behaviour of an endogenously determined interest rate in a world with depletable or recycleable resources. The general properties of such behaviour, as we show, will vary greatly depending on the specific structure of the productive relationships. For simplicity, we shall deal with models including at most one non-depletable commodity. It can represent a medium of exchange for non-depletable private goods in a many-good economy. We shall call this non-depletable good " corn ".

[1] If the marginal cost of extraction is constant, the extraction of a unit of resource today does not affect the cost of extraction in future periods, so that the discounted marginal costs reduce simply to the marginal cost today. If, however, the marginal cost of extraction increases as a function of the quantity already extracted (e.g. if it is necessary to dig deeper into a mine), then the marginal cost of extracting a unit today must be added to the discounted increments to cost in the future due to the extraction of a unit today.

[2] In the previous paper (Weinstein and Zeckhauser [14]) it was argued that the price of such a resource must rise at the rate of interest if extraction and handling were free. But if extraction costs or handling costs are high, then these may dominate the raw resource cost. Then, total price (as distinguished from the residual rent after extraction and handling are subtracted out) may not appear to rise at all.

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It is the only good valued for consumption, and it also serves as an input to its own production process.

The depletable resource we shall call "energy"; its total supply is E^*. The value of energy lies in its contribution to the production of corn. We consider behaviour in a variety of situations. For propositions where rigorous proofs might require tortuous analyses, we present intuitive arguments.

Represent the gross corn output of the economy in period i as Y_i. This output is divided into consumption, C_i, and corn input to the production process, K_i. Energy input in period i is E_i, where maintenance of the energy supply is costless and where the constraint on eternal energy consumption is that

$$\sum_{i=0}^{\infty} E_i \leqq E^*.$$

The production function, which assumes constant technology and labour input, is given by

$$Y_{i+1} = f(E_i, K_i).$$

The population is fixed in size and is assumed to live eternally.

Optimal and market-equilibrium behaviour for this economy will be seen to depend on the level of production that can be achieved with zero input of the depletable resource. The three key possibilities when $E_i = 0$ are as follows:

Case 1. $\inf\{f(0, K_i)/K_i\} = 1+d$ where $d>0$;

Case 2. $\sup\{f(0, K_i)/K_i\} = 1$;

Case 3. $\sup\{f(0, K_i)/K_i\} < 1$;

(where $\inf\{x_i\}$ is the largest number less than or equal, and $\sup\{x_i\}$ is the smallest number greater than or equal, to all possible values of x_i). For all three cases we shall assume that for any K_i there exists an E_i such that $f(E_i, K_i) > K_i$: that the economy can grow if it has sufficient energy. Note that these cases are mutually exclusive. They exhaust the class of production functions that are homogeneous of degree one, but not all production functions.

In Case 1 we shall say that energy is non-vital: such an economy can achieve sustained consumption and growth forever. If the rate of time preference (as applied to resources, not utility) remains above d, it will ultimately set the rate of interest in the economy. Here we shall be interested in situations where time preference, which exerts its influence through constraints on capital accumulation, does not place a floor on capital productivity. Then, if the production function is homogeneous of degree one, it will turn out that the rate of interest in the economy—that is, the own rate of return for corn—will converge to d. If energy is vastly abundant relative to corn, it may permit the rate of return on corn to remain above d for a significant period of time. But in the long run, as the energy supply runs out, the base rate d will predominate. Along the equilibrium path, the price of energy (relative to corn) will rise at the current own rate of return of corn. In the long run, that rate—which is the market rate of interest—will approach d. Say corn's own rate of productivity in the long-run equilibrium is 1.1. Then the price of energy (i.e. its marginal productivity) will rise by 10 per cent per period. This, then, is the natural generalization of the main result in our previous paper [14], with the endogenously determined rate d corresponding to the fixed rate r in our model.

In Case 2 energy is necessary for growth, but it is not necessary for sustaining the capital stock if consumption is zero.[1] In other words, there is costless storage of corn.

[1] We could tolerate zero consumption if we interpret the production function as having the corn necessary to sustain the population already subtracted out. The output is then output net of subsistence.

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Robert Solow [13] has treated this model in a most elegant fashion, with the understanding that capital (corn) is at least maintained from period to period; it is used as an input along with energy (in our terms) to produce a net increase in the supply of corn. Solow allows labour as a productive input as well, but with the labour supply fixed,[1] this merely adds a multiplicative constant to the production function.

One of the models Solow considers is of the form

$$f(E_i, K_i) = K_i + g(E_i, K_i) = K_i + L^{1-a-b} K_i^a E_i^b, \qquad \ldots(1)$$

where g is homogeneous of degree $a+b<1$ (viewing L as constant). He shows that if $a>b$, then a level stream of consumption $C_i = C_0$ can be maintained forever. We shall return to the implications of that result presently.

Case 3 relates to a variety of real world situations. It is the case where the presence of the depletable resource most dramatically affects the evolution—and demise—of the economy. It should be noted that in Case 3 it is not possible to maintain a level consumption stream indefinitely.

There are a variety of situations that might lead to the Case 3 model. A range of Case 2 models, modified by a constant depreciation rate, will take this form. The natural amended form of the Solow model would be

$$f(E_i, K_i) = sK_i + g(E_i, K_i),$$

where $s<1$. A special class of situations where Case 3 applies has $s = 0$ in the above equation and includes all production functions such that some positive amount of energy is required to maintain the capital stock. For example,

$$f(E_i, K_i) = K_i^\alpha E_i^{1-\alpha}$$

is of this type. If $E_i = 0$, then the economy ceases to produce as the corn supply is annihilated.

In all Case 3 and some Case 2 situations,[2] we are placed in a situation where there is an upper bound on the total amount of consumption that will ever be available. This bound will be a function of the energy supply. In such situations, energy might be termed " vital to the total consumption level ". It would seem that these are the types of situations environmentalists have in mind when they sound the depletable-resource tocsin. (In certain cases, specific consumption patterns will be required even to reach the energy-limited bound.)

We shall consider the optimal and equilibrium path for production and consumption, first in general terms and then with reference to examples in Cases 2 and 3, where the properties of the equilibrium path are of particular interest. We first discuss optimality under centralized control. Then we will argue that the optimality conditions of this path will be realized given the conditions of a competitive market.

Consider an objective function of the general form $u(C_0, C_1, \ldots)$. As a society, we wish to choose the sequence $\{C_i\}$ to maximize u subject to the production constraint

$$Y_{i+1} = f(E_i, K_i), \quad i = 0, 1, \ldots, \qquad \ldots(2)$$

the conservation of corn constraint

$$C_i + K_i = Y_i, \qquad i = 0, 1, \ldots, \qquad \ldots(3)$$

the energy constraint

$$\sum_{i=0}^{\infty} E_i \leqq E^*, \qquad \ldots(4)$$

the initial stock Y_0, and non-negativity restrictions on all variables.

1 As he assumes in his zero-population-growth model.

2 For example, the situation in equation (1) with $a<b$.

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The intertemporal efficiency conditions for the optimal path are given as follows. Let

$$f_1^i = \partial f(E_i, K_i)/\partial E_i;\ f_2^i = \partial f(E_i, K_i)/\partial K_i;\ \text{and } u_i = \partial u/\partial C_i.$$

Then

$$u_i/u_{i+1} = f_2^i \quad \ldots(5)$$

and

$$u_i/u_{i+1} = \frac{f_1^i}{f_1^{i-1}}. \quad \ldots(6)$$

These, together with (2) and (3) can be used to generate Y_{i+1}, C_{i+1}, K_{i+1}, and E_{i+1}, from Y_i, C_i, K_i and E_i. To compute the efficient initial energy input and appropriate accompanying division of the corn stock may be more difficult. It will involve forward looking calculations employing the energy constraint equation (4).

These optimality conditions can be interpreted as follows. Equation (5) says that the marginal rate of substitution for consumption between periods must equal the own rate of return on corn. This is a familiar result dating back to Fisher [5]. Combining (5) and (6) we get the result that

$$f_1^{i+1}/f_1^i = f_2^{i+1}. \quad \ldots(7)$$

Recognizing f_1^i as the marginal productivity of energy and f_2^i as the own rate of return of corn (i.e. the market interest rate plus one), we can interpret (7) as the condition that the price of energy (i.e. its marginal productivity) rises at the rate of interest.

The Competitive Equilibrium and the Optimal Path [1]

These efficiency conditions on consumption rely on the specification of an objective function, a function that hardly exists in the real world. But if markets for depletable and recycleable resources are competitive, prices, excess demands, and other economic indicators will behave as if there were some central processing system maximizing some global welfare function.[2] An intuitive demonstration of the existence of the competitive equilibrium would start by specifying each (immortal) individual's endowments of the two goods, corn and energy. All trades would be carried out at the outset. The price of energy would be constant in each period. The relative price for corn in each period would be determined through traditional tatonnement processes.

There are three possible complexities. If the number of periods stretches to infinity, a more sophisticated demonstration of competitive equilibrium, such as in Bewley [2], may be required. Second, lifetimes are finite; a competitive equilibrium exists because they also overlap. Individuals work in the first portion of their lives to gain claim over the existing resource stock, then sell off their assets as death approaches. The upward slope of their first-save-then-dissave consumption stream would flatten, (and the own interest rate for corn would be lowered), if their lives were lengthened.[3] Finally, as Robert Solow reminded us, given inherent restrictions on the vision of market participants, and the absence of appropriate futures markets, these eternally impacting markets for resources may encounter particular problems of stability. From time to time, they will encounter significant readjustments as expectations are altered, or new information on resource

[1] Malinvaud [8] analyses a market model where the interest rate is determined endogenously and demonstrates the efficiency of the result. His model, though, assumes a fixed supply of natural resources E_i in each period rather than a stock E^* that can be allocated freely between periods.

[2] The method of argument to prove this proposition would treat period consumptions in a manner parallel to the treatment of individuals' endowments in the competitive equilibrium proof by Negishi [9]. By way of example, if each individual j has a utility function $\Sigma_t(1+r)^{-t} \log C_{tj}$ then the market outcome would maximize the social objective function $\Sigma_t(1+r)^{-t} \log C_t$ where $C_t = \Sigma_t C_{tj}$.

[3] See Zeckhauser [15] and Samuelson [12].

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supplies becomes available. But the grand sweep of economic behaviour will follow the patterns described here.

Optimal Paths for Specific Models

It is instructive, before moving on to recycling, to consider the optimal paths with various Case 2 and Case 3 models, and under various objective functions. Consider first a Case 3 model with

$$f(E_i, K_i) = 2E_i^{0\cdot5}K_i^{0\cdot5}. \qquad \text{...(8)}$$

Furthermore, let

$$u(C_0, C_1, \ldots) = \sum_{i=0}^{\infty} (1+r)^{-i} \log C_i, \qquad \text{...(9)}$$

where r is some rate for discounting utility.[1]

It turns out that the optimal path is characterized by

$$E_{i+1} = E_i/(1+r); \qquad \text{...(10)}$$

$$C_{i+1} = C_i f_2^i/(1+r); \qquad \text{...(11)}$$

and

$$f_2^{i+1} = (f_2^i)^{0\cdot5}. \qquad \text{...(12)}$$

Thus, energy use falls geometrically and consumption falls geometrically in the long run as the rate of interest $f_2^i - 1$ approaches zero.[2]

Figure 1 shows the path of consumption and total output for Case 3. The sequence of energy use is always of the form $E_0 = (1-1/(1+r))E^*$ and $E_{i+1} = E_i/(1+r)$. Note

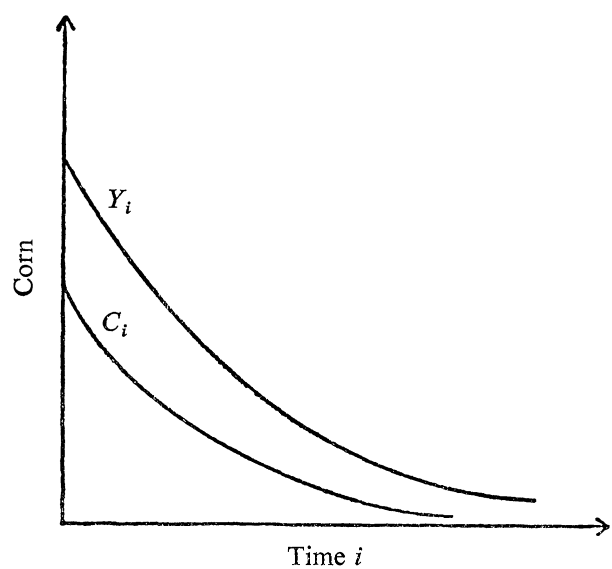

FIGURE 1

Output and consumption: Case 3 with discounted utility

[1] The behavioural assumptions underlying discounting of utility are rather stringent, but since it is customary to consider this form for utility in the literature we shall use it here.

[2] The convergence of f_2^i to unity is implied by (12). If $Y_0 = 2rE^*$, then f_2^i starts at unity and stays there. If $Y_0 > 2rE^*$, then f_2^i approaches unity from below; if $Y_0 < 2rE^*$, then f_2^i approaches unity from above.

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that in Case 3 consumption and output must approach zero at a rate approaching the constant $1/1+r$. Also note that if $r = 0$, the solution is indeterminate.[1]

Solow has employed an objective function different from (9) for optimizing social welfare. His objective function, derived from Rawls [11], is

$$u(C_0, C_1, \dots) = \inf_i \{C_i\}. \qquad \dots(13)$$

Essentially, society is no better off than its poorest generation. Now with a Case 3 production function it is clear that the highest possible value of this function is zero. No matter how we allocate resources, we cannot escape the fact that consumption must drop to zero sooner or later. A level consumption stream cannot be maintained.

Now consider a Case 2 model, like Solow's, with

$$Y_{i+1} = K_i + 10E_i^{0\cdot 2}K_i^{0\cdot 4}. \qquad \dots(14)$$

First, we consider the optimal path under our objective function (9) for positive and zero values [2] of r. Then we consider the Rawls criterion (13).

A computer was used to generate the optimal path under objective function (9) with $r = 0{\cdot}25$, $Y_0 = 75$, and $E^* = 103$. It turns out that energy use falls at a rate approaching a constant geometric factor of $1+r = 1{\cdot}25$. Meanwhile, the rate of interest (which is the own rate of return on corn) approaches a steady state value of 0·17. Consumption starts at about 36·8 rises for a while to a maximum of 96·6, and then falls after the rate of interest falls below the rate of discount for utility (see equation (11)). Figure 2 displays the path of energy consumption.

The solution to Solow's problem with the Rawls objective function (13) is, of course, a steady consumption stream which, for the production function (14) with Y_0 and E^* as above, is $C_0 = 57{\cdot}2$. This is the maximum sustainable level of consumption. Figures 2 and 3, drawn to scale, summarize the calculated results for the two examples given. We discover that the nature of the objective function as well as the nature of productive relationships in the economy can significantly affect patterns of use for depletable resources.

It would be presumptuous and a bit foolhardy to suggest which of our three models of production best describes the real world. In some instances one model will yield greater insights; sometimes, depending on the resource and time period under examination, another. A number of observers suggest that Case 2 or 3 forces are on the verge of

[1] To further an intuitive understanding of the problem, it is instructive to compare behaviour along the early portions of the optimal path for given initial endowments, with the asymptotic path. If we start off the asymptotic path, it is because either corn or energy is in what we might call " relative abundance ". (This problem does not arise in Case 2 situations because the corn/energy input ratio varies systematically along the optimal path.)

Assume for the moment that all the energy stock could be combined with the amount of corn that maximized the net addition to the initial corn pile, NA. Thus

$$NA = \max_Y f(E_0, Y) - Y.$$

Consider then an alternative economy that starts off with $Y_0 + NA$ corn, no energy, and with free storage. Its optimization task would be a straightforward " how fast should we deplete the stock " problem.

This deplete-the-stock economy would be at least as well off as the actual economy, perhaps better. It will be better if the optimal path for the actual economy diverges from its optimal path. This would happen if initial corn were either in very short or very long supply relative to energy. In the very-short case, consumption will have to be held down somewhat for a while for there is insufficient corn to combine with energy in the net-addition-maximizing ratio. (Over time, the price of energy in terms of corn will increase.) In the very-long relative supply situation there may be a problem if storage is not free. Energy will be combined with more than the NA-maximizing amount of corn at the outset. The price of energy will fall in terms of corn.

In the real world, economic forces seem to be pushing up energy prices in terms of reproducible resources. If Case 3 were an accurate description of the world, this would suggest that energy is in relatively greater abundance than it would be along an asymptotic path. It would also suggest that over time the long run rate of increase of the energy price will diminish, not accelerate.

This discussion may illustrate that at least for Cases 2 and 3, energy is no more depletable than is corn; it too can yield only a finite stock over time.

[2] Note that when $r = 0$, (9) is ordinally equivalent to $u(C_0, C_1, \dots) = \prod_t C_t$.

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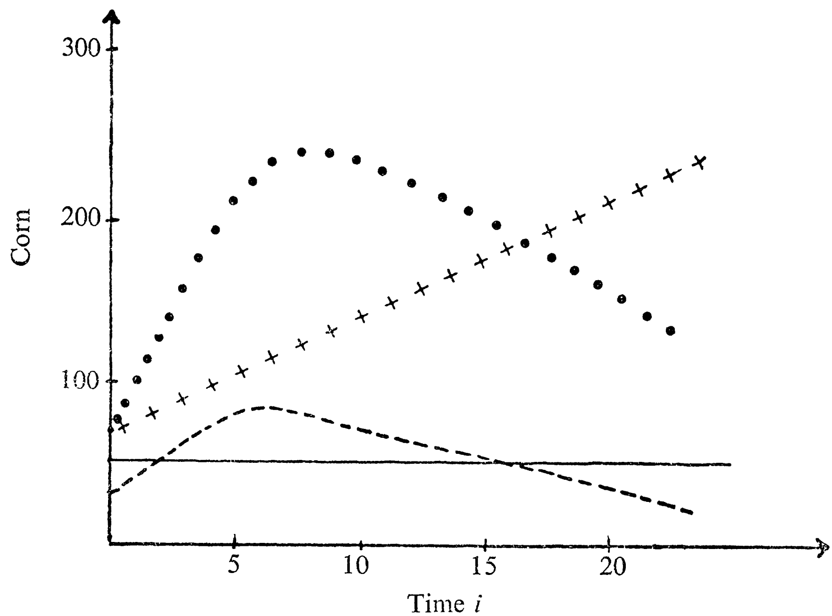

FIGURE 2

Consumption and total corn output. - - - Consumption: discounted utility. ——— Consumption: Rawls utility. · · · Output: discounted utility. + + + Output: Rawls utility.

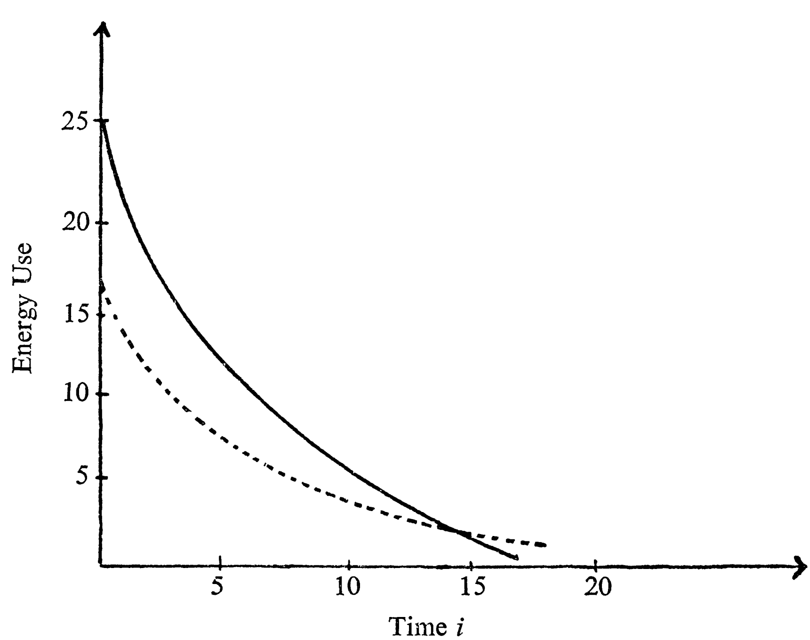

FIGURE 3

Energy use. ———Energy: discounted utility. - - - - Energy: Rawls utility.

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predominating in this half of the twentieth century; i.e. emerging restrictions on resource supplies will change the performance of capital markets, and limit prospects for growth in the capital stock. Perhaps, quite to the contrary, prospective new energy technologies will lead to a world where natural resource limitations play a diminished role. Our analysis assists us in choosing among these models to the extent that it predicts implications and consequences that can be observed more readily than the structures of the models *per se*.

The remainder of this paper, like most discussions of multi-period efficiency from the time of Irving Fisher, assumes the exogenous specification of an interest rate between each pair of successive periods. We have just shown how such rates could be determined endogenously. If energy, or any other depletable resource, is a resource of great consequence, and if there is not a floor rate of own productivity for reproducible resources above zero, then indeed this rate may vary. In such cases, what we have labelled $1+r$ should be recomputed as p_i/p_{i+1} where p_j gives the price of corn in period j relative to some base period.

It is interesting to reiterate, before we move on, that where energy is vital to the total consumption level it will eventually influence the discount rate and push it towards its limit of zero. For those particular situations, the subject of much discussion but in our opinion of little real world consequence, the analysis that follows is of diminished relevance.

3. NON-DEPLETABLE RESOURCES WITH ZERO RECYCLING COST

A model with zero recycling cost provides a good introduction to use patterns in a world with recycleable resources. It is also useful in defining notation and terminology for later more complex models. Assume for the moment that extraction costs are zero, or that the total supply has already been extracted.[1] A fixed supply of this resource, Q, will be available for use in each period. Consumption occurs in discrete time periods; at the end of each, the commodity is immediately ready for reuse.

The socially optimal consumption stream is trivial to determine. Given no depletion, then assuming the good's value never falls below zero, the total quantity Q should be in use in all periods. Individuals with the highest marginal willingness to pay for the use of the good in any period should possess it in that period.[2] This outcome will be achieved by setting a *rental* charge x_t^* in each period t such that the amount Q is just demanded. With the demand curve given by $x_t = d_t(q_t)$, the required relationship is $x_t^* = d_t(Q)$. In a perfectly competitive economy, each rental period serves in isolation as a static market. The market price will vary each period to just clear the market; the resource will be efficiently allocated.

The *selling price* of the commodity in any period, p_t, will be the value returned for owning it in perpetuity. It is thus the discounted sum of present and future rentals,

$$p_t = \sum_{i=t}^{\infty} (1+r)^{t-i} x_i^*. \qquad \ldots(15)$$

It is a direct consequence of (15) that the price satisfies the recursive relation

$$p_{t+1} = (p_t - x_t^*)(1+r). \qquad \ldots(16)$$

Intuitively, this says that the price at which an owner is willing to sell his diamond next year is equal to this year's price, *less* the rental he could collect this year if he maintained ownership, multiplied by the productivity of other forms of capital, $1+r$.

1 The models in Sections 5 and 6 cover as special cases situations where extraction costs are zero but where recycling costs are non-zero.

2 The numeraire to be maximized, as before, is discounted surplus

$$S = \sum_{t=0}^{\infty} (1+r)^{-t} \int_0^{q_t} d_t(\xi_t) d\xi_t.$$

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4. PARTIALLY DEPLETABLE RESOURCES WITH ZERO RECYCLING COST

The possibility of partial depletion of a resource each time it is used lies between the extremes of our previous paper and the model just considered. Chemical catalysts employed in production processes may exhibit this property; only a fraction of the original supply is available for reuse when the process is completed.[1]

Let f, $(0 \leqq f < 1)$, represent the fraction of the consumed good that is available for use in the next period. The strictly depletable resource model is the empirically important special case where $f = 0$. Allowing for other values of f, the total effective supply is

$$Q+Qf+Qf^2+\ldots+\ldots = Q/(1-f).$$

If half of the consumed good is available in the period after use, for example, the supply is twice what it would be were it strictly depletable.

The rental transaction in the partly depletable model consists of a one-period loan, with f times the original quantity loaned returned at the end of the period. In some very real sense, then, this is a model of strict depletion, where the total supply is $Q/(1-f)$, and each period's rental of one unit is equivalent to a strict depletion of $1-f$ units. Intuition, educated by the results of the strictly depletable model, suggests that the optimality and equilibrium results would have rentals (i.e. the rights to deplete) rise by the factor $1+r$ per period. This will indeed be the result, unless corner solutions come into play.

The possibility of corner solutions arises because the total effective supply is not available at once for consumption in any period. Suppose that in period 0 the rental x_0, that would pertain were the resource strictly depletable with supply $Q/(1-f)$, would result in demand in excess of Q. Then there would be a shortage that could not arise in the " equivalent model ". As might be expected, when there are shortages of this type, rentals temporarily rise at a rate greater than r; this is followed by a new lower level of discounted rentals. These then rise by $1+r$ until another corner solution is met.

The socially optimal consumption pattern is easily derived as follows. Our objective is to choose $\{q_t\}$ to maximize

$$S = \sum_{t=0}^{\infty} (1+r)^{-t} \int_0^{q_t} d_t(\xi_t)d\xi_t, \qquad \ldots(17)$$

subject to the constraints

$$\sum_{t=0}^{\infty} q_t \leqq Q/1-f, \qquad \ldots(18)$$

$$\begin{cases} q_t \leqq Q-(1-f)\sum_{i=0}^{t-1} q_i, \quad (t=1, 2, \ldots), \\ q_0 \leqq Q, \end{cases} \qquad \ldots(19)$$

and, of course, $q_t \geqq 0$ for all t. Constraint (19) says that the amount consumed in any period cannot exceed the supply *on hand*. Constraint (18) is really redundant but is included because its shadow price has a useful interpretation. Adjoin (18) and (19) to (17) by multipliers λ and $\{\mu_t\}$ respectively. Differentiating the Lagrangean with respect to q_t and setting the result equal to zero yields the maximizing conditions

$$x_t = (1+r)^t \left(\lambda+\mu_t+(1-f)\sum_{i=t+1}^{\infty} \mu_i\right), \qquad \ldots(20)$$

where $x_t = d_t(q_t)$ is the rental in period t.

[1] Stamped metals are a good illustration of this model in combination with some to be considered later. The unstamped fractions of sheets can be melted and rerolled into new sheets for later stamping. The two deviations from this model are that rerolling entails a positive recycling cost, and that there is the possibility that the stamped metal will some day be recovered.

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Condition (20) can be easily interpreted. If the constraints (19) are never binding (i.e. if we never run into temporary shortages because the residual f is not available for instantaneous reuse), then all $\mu_t = 0$, and $x_t = (1+r)^t\lambda = (1+r)^t x_0$ as in the model of perfectly exhaustible resources. Now suppose that there exists a k such that (19) is binding for $t = k$ but is not binding for all other t. Then

$$x_t = \begin{cases} (1+r)^t(\lambda+(1-f)\mu_k) & \text{for } t<k \\ (1+r)^t(\lambda+\mu_k) & \text{for } t=k \\ (1+r)^t\lambda & \text{for } t>k. \end{cases} \qquad \ldots(21)$$

The rental rises at the rate r up through period $(k-1)$; it jumps a bit in period k (when the period supply constraint is binding); and then it drops to a lower present value than before.

Now, let us verify that a competitive market will give this same allocation. In order for a non-zero quantity to be supplied in each period, the present value of the stream of rentals starting in each period must be at least as great as the present value of rentals starting in all successive periods. Otherwise, the suppliers will withhold the commodity from the market until the present value of rentals is maximized. The present value of rentals, if renting is started in period t, is given by

$$v_t = \sum_{i=t}^{\infty} (1+r)^{-i} f^{i-t} x_i. \qquad \ldots(22)$$

If these are to be equal for all periods (to some common value π_0), then we must have

$$\pi_0 = f\pi_0 + (1+r)^{-t}x_t$$

for all t, so that

$$x_t = (1+r)^t(1-f)\pi_0,$$

and the rental grows at the rate r. Note that we can interpret

$$\pi_0 = \lambda/(1-f)$$

as the value, or selling *price*, of a unit of the commodity in period 0, and

$$\pi_t = \pi_0(1+r)^t = v_t(1+r)^t$$

as the price in period t. Thus, the *price*, as well as the *rental*, grows at the rate r.

It was noted that in order for the market to clear it is only necessary that the sequence of present values (22) be non-increasing. If the sequence *decreases* in any period, it can be easily shown using (21) that this indicates a situation where $\mu_{t-1} > 0$. In such a situation, the sequence $\{v_t\}$ in (22) will be level up through the period $t-1$ where $\mu_{t-1} > 0$, and will then drop. Suppliers would have liked to rent out more in period $t-1$ and less in period t, but unfortunately their supply was not intertemporally fungible. Physically constrained, they rent out their total supply in period $t-1$, as the positive shadow price μ_{t-1} would suggest.

5. RECYCLEABLE RESOURCES WITH ZERO EXTRACTION COSTS

Suppose as before that there is a resource in fixed supply Q, and that extraction costs are zero. Now, however, suppose that the resource, once used, can be recycled at a cost c. We shall assume for simplicity that recycling is instantaneous and occurs at the beginning of any period in which the recycled good is to be used. Consumption takes exactly one period.[1]

[1] Alternatively, we might assume that consumption is instantaneous at the end of each period and that recycling is a process that takes one period. Things get a bit more messy, but the analytic structure remains basically the same, if both consumption and recycling take positive fractions, say $\frac{1}{3}$ and $\frac{2}{3}$ of the period. Our formulation in the text enables us to define a period to be that length of time between initial consumption and availability for next consumption. With heavy metals, this might be ten years.

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We would expect the market equilibrium allocation to proceed as follows. Since extraction is costless, while recycling has a positive cost, we would expect the total supply Q to be used up before any recycling begins. During the time that the supply is being used for the first time, the rental would be rising at the rate of interest r; otherwise none of the good would be provided in periods when

$$(1+r)^{-t}x_t < \max_t (1+r)^{-t}x_t.$$

In the periods after the supply is used up, demand is met entirely by recycling. The rental in those periods is equal to the marginal cost c, unless demand at that rental exceeds Q. If $d_t^{-1}(c) > Q$, then the price is above marginal cost c and suppliers obtain a profit.

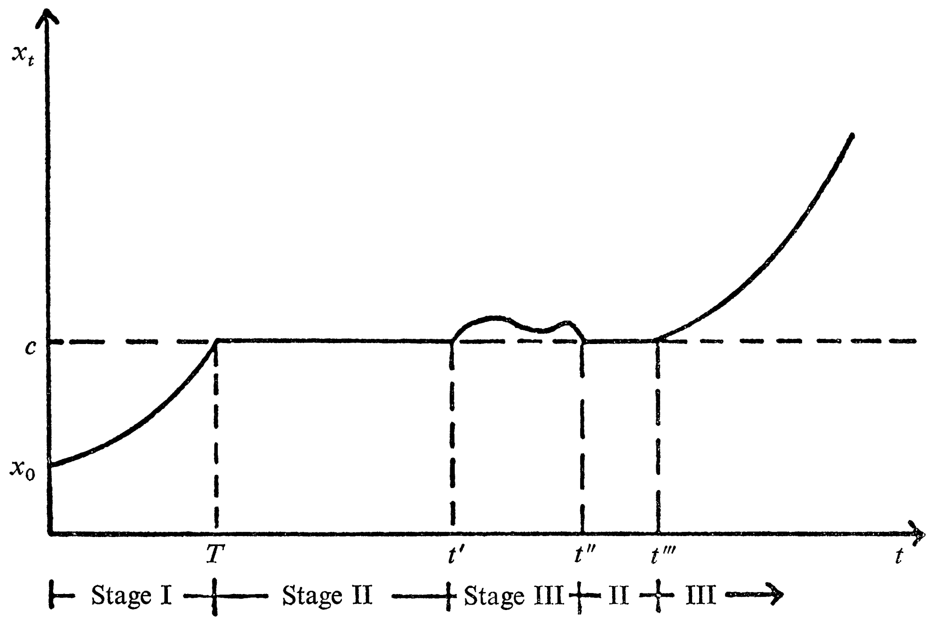

FIGURE 4
The three stages of resource use

If the demand over time is stable or slowly increasing, we would expect the rental to equal c for several periods, and then to rise above c when the demand at c increases above the supply Q.

It is clear that the rental in the last period in which the unrecycled resource is being used up must lie between $c/(1+r)$ and c. Otherwise suppliers, anticipating a rental increase at a rate exceeding r, would withhold the commodity from the market. Therefore, the rental charge for the unrecycled resource rises at the rate r until (1) the supply Q is exhausted and (2) the rental lies between $c/(1+r)$ and c. These two conditions determine a unique period T during which the supply is finally exhausted. Thereafter, the rental equals marginal cost c in all periods when $d_t^{-1}(c) \leq Q$, and it exceeds marginal cost c in all periods when $d_t^{-1} > Q$.

The scenario proceeds in three stages. In Stage I, we are still mining the natural resource. During this stage, the rental rises at the rate r. At the end of this stage, the rental is within a factor of $1/(1+r)$ of the cost of recycling, c. In Stage II, demand is met entirely by recycling and the rental is equal to marginal cost c. During Stage II, demand at the rental c can be met by the supply Q available for recycling in each period. In Stage III, demand at the rental c cannot be met by the supply. The rental rises above c to clear the

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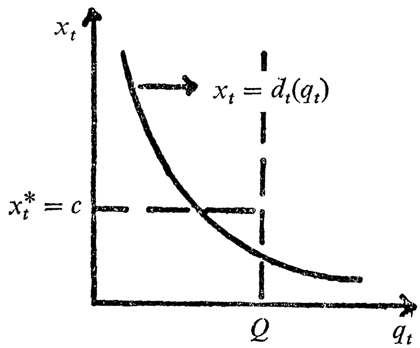
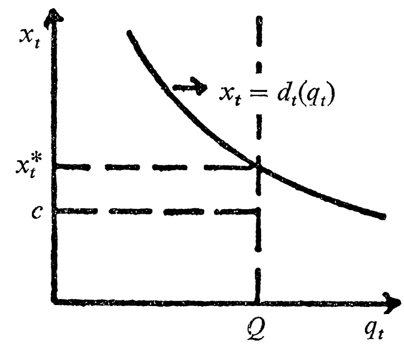
market. Note that if demand contracts at any point in time, it may be possible to move from Stage III back into Stage II. It is never possible to move back into Stage I, since the resource in its natural state is gone forever once Stage II is entered.

Figure 4 displays price as a function of time for a typical scenario. Note that Stage II is entered at period T, and that Stage III is entered at period t'. Note also that it is possible to re-enter Stage II at t'', and to re-enter Stage III at t'''. The demand curves in typical Stage II and Stage III periods are shown in Figure 5. The equilibrium rental x_t^* in Stage II is equal to the marginal cost of recycling, c; the rental x_t^* in Stage III exceeds c.



a. Stage II



b. Stage III

FIGURE 5
Typical Stage II and Stage III demand curves

As we are by now accustomed to expect, the socially optimal consumption stream corresponds precisely to the market equilibrium. The proof, using the Kuhn-Tucker conditions, is instructive and is given here in its entirety.

Define q_t as the amount of previously unused resource used in period t, and define r_t as the amount of recycled resource used in period t. Our objective is to maximize

$$S = \sum_{t=0}^{\infty} (1+r)^{-t} \left[\int_0^{q_t+r_t} d_t(\xi_t) d\xi_t - r_t c \right] \qquad \text{...(23)}$$

subject to the constraints

$$\sum_{t=0}^{\infty} q_t \leqq Q, \qquad \text{...(24)}$$

$$\begin{cases} r_t \leqq \sum_{i=0}^{t-1} q_i \quad (t \geqq 1), \\ r_0 = 0 \end{cases} \qquad \text{...(25)}$$

$$q_t \geqq 0 \quad (\text{all } t), \qquad \text{...(26}$$

and

$$r_t \geqq 0 \quad (\text{all } t). \qquad \text{...(27)}$$

Constraint (25) says that the amount recycled in period t cannot exceed the amount of previously used natural resource.

Adjoin (24) by a multiplier λ, (25) by multipliers $\{\rho_t\}$, (26) by multipliers $\{v_t\}$, and (27) by multipliers $\{\mu_t\}$, obtaining a Lagrangean

$$L = S - \lambda \left(\sum_{t=0}^{\infty} q_t - Q \right) + \sum_{t=0}^{\infty} \mu_t r_t + \sum_{t=0}^{\infty} v_t q_t - \sum_{t=0}^{\infty} \rho_t \left(r_t - \sum_{i=0}^{t-1} q_i \right). \qquad \text{...(28)}$$

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Differentiating (28) with respect to q_t and r_t, we have the optimality conditions

$$(1+r)^{-t}x_t-\lambda+\nu_t+\sum_{i=t+1}^{\infty}\rho_i=0 \qquad \text{...(29}a\text{)}$$

and

$$(1+r)^{-t}x_t-(1+r)^{-t}c+\mu_t-\rho_t=0, \qquad \text{...(29}b\text{)}$$

where $x_t = d_t(q_t+r_t)$ is the rental in period t.

We now apply the Kuhn-Tucker conditions by considering separately the cases $\mu_t = \nu_t = 0$ ($r_t>0$, $q_t>0$), where $\mu_t>0$ and $\nu_t = 0$ ($r_t = 0$, $q_t>0$), and where $\mu_t = 0$ and $\nu_t>0$ ($r_t>0$, $q_t = 0$). Each of these cases has an interpretation in terms of the three-stage consumption model.[1]

Case A: $\mu_t = \nu_t = 0$.
By (29*a*) and (29*b*)

$$(1+r)^{-t}x_t-\lambda+\sum_{i=t+1}^{\infty}\rho_i=(1+r)^{-t}(x_t-c)-\rho_t=0, \qquad \text{...(30)}$$

so that

$$\lambda=(1+r)^{-t}c+\sum_{i=t}^{\infty}\rho_i. \qquad \text{...(31)}$$

The expression on the right-hand side of (31) is seen to be *strictly* decreasing as a function of t, since the first term is strictly decreasing and the second term is non-increasing (because the multipliers ρ_t are all non-negative). Since λ is a constant, equation (31), and therefore Case A, can occur in *at most* one period. Call such a period T. If a Case A situation ever does exist then, by (30),

$$\lambda=(1+r)^{-T}x_T+\sum_{i=T+1}^{\infty}\rho_i. \qquad \text{...(32)}$$

Furthermore, by (30),

$$x_T=c+\rho_T(1+r)^T \geqq c. \qquad \text{...(33)}$$

Thus, the rental in period T exceeds the cost of recycling, c, unless $\rho_T = 0$, in which case the rental is equal to c.

In terms of the three stages, this period T when both initial use (q_t) and recycling (r_t) are occurring is the boundary between Stages I and II. If $\rho_T = 0$ (so that there is not a binding shortage of recycleable resource) the rental x_T in this period is equal to c. If $\rho_T>0$, then this describes a situation where Stage II is bypassed and Stage III is entered immediately. The multiplier ρ_T is interpreted as the present value of surplus to the supplier in period T due to a stage III excess demand situation. This interpretation will become clear in the discussion of Case C ($q_t = 0$, $r_t>0$).

Case B: $\mu_t>0$, $\nu_t = 0$
For periods in which $\mu_t>0$, the Kuhn-Tucker condition says that $r_t = 0$. Since $r_t = 0$, constraint (25) canot be binding, and we must have $\rho_t = 0$.[2] Therefore, from (29)

$$(1+r)^{-t}x_t-\lambda+\sum_{i=t+1}^{\infty}\rho_i=(1+r)^{-t}(x_t-c)+\mu_t=0. \qquad \text{...(34)}$$

[1] The case $\mu_t>0$ and $\nu_t>0$ ($r_t = q_t = 0$) is not terribly interesting and is not discussed. Such a period of zero consumption can never occur in Stage I, and can occur in Stage II only if the demand curve cuts the vertical axis below c.

[2] Except possibly in period 0. Such a situation would reflect a Stage III excess demand situation in the very first period.

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Observe that $x_t = c - \mu_t(1+r)^t < c$ in these periods. Also from (34) we have

$$x_t = \left(\lambda - \sum_{i=t+1}^{\infty} \rho_i\right)(1+r)^t, \qquad \ldots(35)$$

so that the rental x_t is increasing at the rate r as long as $\rho_t = 0$.

Suppose for now that there exists a period T satisfying the conditions of Case A. Then substituting (32) into (35),

$$x_t = \left[(1+r)^{-T}x_T - \sum_{i=t+1}^{T} \rho_i\right](1+r)^t. \qquad \ldots(36)$$

But as long as Case B holds, $\rho_t = 0$. Furthermore, Case B holds for all $t<T$ since the rental x_t is increasing and (by (36)) is always less than x_T. Therefore, (36) reduces to

$$x_t = [(1+r)^{-T}x_T - \rho_T](1+r)^t.$$

Using equality (33), we have

$$x_t = c(1+r)^{t-T} \qquad \ldots(37)$$

for all $t<T$. Thus

$$x_0 = c(1+r)^{-T}. \qquad \ldots(38)$$

Substituting (33) and (38) into (32),

$$\lambda = x_0 + \sum_{i=T}^{\infty} \rho_i. \qquad \ldots(39)$$

Equations (37) and (38) show that the rental is growing at the rate r up until period T, at which point $x_T = c$. This fits the description of Stage I of our general scenario.

Now suppose that there does *not* exist a period T satisfying the conditions of Case A. We know that (35) still holds, and ρ_t still vanishes for all periods falling under this heading. Here define T as the last period in which $q_t > 0$ ($\mu_t = 0$). Then by (35)

$$x_T = \left(\lambda - \sum_{i=T+1}^{\infty} \rho_i\right)(1+r)^T. \qquad \ldots(40)$$

Hence, we conclude from (35) that

$$x_0 = \lambda - \sum_{i=T+1}^{\infty} \rho_i \qquad \ldots(41)$$

and

$$x_t = x_T(1+r)^{t-T} = x_0(1+r)^t \qquad \ldots(42)$$

as before. We still have a Stage I situation; rentals grow at the rate r up until period T when the resource is exhausted. Unlike the situation described by (37)-(38), however, there is no recycling in period T (it starts in period $T+1$) and the rental x_T is strictly less than c. In fact, it is easy to show that the rental x_T must lie between $c/1+r$ and c as claimed earlier.[1]

It is not difficult to see how the cutoff period T between Stages I and II is determined. In the process, we also discover whether there exists a period T in which $x_T = c$ and in which both q_T and r_T are nonzero, or whether, instead, the last period of extraction, T,

[1] See footnote 1, p. 82 for a proof; it depends on some results from Case C.

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is characterized by the inequalities $c/(1+r) \leqq x_T \leqq c$ and $r_T > 0$. Both T and the initial rental x_0 are determined uniquely as follows. Find the unique period T' such that

$$\sum_{t=0}^{T'-1} d_t^{-1}[(1+r)^{t-T'+1}c] < Q,$$

but

$$\sum_{t=0}^{T'} d_t^{-1}[(1+r)^{t-T'}c] \geqq Q.$$

If, in addition,

$$\sum_{t=0}^{T'-1} d_t^{-1}[(1+r)^{t-T'}c] \leqq Q,$$

then $T = T'$ and $x_T = c$. If, however,

$$\sum_{t=0}^{T'-1} d_t^{-1}[(1+r)^{t-T'}c] > Q,$$

then $T = T'-1$, and $c/(1+r) \leqq x_T \leqq c$.

Case C: $\mu_t = 0$, $\nu_t > 0$

Now, finally, let us turn to Case C which embodies both Stages II and III of the consumption scenario. From (29*b*), we see that

$$x_t = c + \rho_t(1+r)^t \geqq c; \qquad \text{...(43)}$$

this is the same inequality that held in Case A.[1] When $\rho_t = 0$, then $x_t = c$ and we are in Stage II. The constraint on recycling is not binding. When $\rho_t > 0$, then $x_t > c$ and we are in Stage III. The optimality proof is now complete.

Before leaving this model with zero extraction costs, it is useful to interpret the multiplier λ. From (39) and (41), it is seen that in all cases

$$\lambda = x_0 + \sum_{i=T}^{\infty} \rho_i.$$

We can interpret λ as the value, or selling price, of a unit of the resource in period 0. This value is the sum of a unit's worth as an untapped resource in Stage I (x_0), and its surplus worth in the periods of Stage III, i.e. where there is excess demand ($\rho_t > 0$). There is no surplus to be accrued in Stage II ($\rho_t = 0$). The price in period t is given by

$$\pi_t = \begin{cases} x_0 + \sum_{i=T}^{\infty} \rho_i & \text{for } t \leqq T \\ \sum_{i=t}^{\infty} \rho_i & \text{for } t > T. \end{cases}$$

[1] We can now give the proof that $c/(1+r) \leqq x_T \leqq c$ as claimed above. By (34), $x_T = c - \mu_T(1+r)^T \leqq c$ since $\mu_T \geqq 0$. To get the other inequality, let $t = T+1$ in equation (29*a*) and substitute $\lambda = x_0 + \Sigma_T^\infty \rho_i$ from (39). Then

$$(1+r)^{-(T+1)} - \left(x_0 + \sum_{t=T+1}^{\infty} \rho_i\right) + \nu_{T+1} + \sum_{t=T+2} \rho_i = 0;$$

simplifying and substituting $x_0 = (1+r)^{-T}x_T$, we get

$$(1+r)^{-(T+1)}x_{T+1} - (1+r)^{-T}x_T + \nu_{T+1} - \rho_{T+1} = 0.$$

Substituting (33) we have

$$(1+r)^{-(T+1)}c - (1+r)^{-T}x_T + \nu_{T+1} = 0.$$

Since $\nu_{T+1} \geqq 0$, this implies that $x_T \geqq c/(1+r)$ as claimed.

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6. RECYCLEABLE RESOURCES WITH CONSTANT MARGINAL EXTRACTION COSTS

We now generalize the model with recycling to allow for non-zero costs of extraction of the resource from its natural state. Assume for now that the marginal cost of extraction is constant at e, and that the recycling cost is c. Since the character of the market equilibrium (which is also efficient) differs considerably in the three cases $c>e$, $c=e$, and $c<e$, these three cases are discussed separately. For each case, the nature of the market equilibrium is described, and a demonstration of the optimality of the equilibrium is then given.

Since we shall need to rely on the mathematical formulation of the optimization problem in all three cases, it will be stated here in full generality. We wish to select $\{q_t\}$ and $\{r_t\}$ to maximize

$$S = \sum_{t=0}^{\infty} (1+r)^{-t}\left[\int_0^{q_t+r_t} d_t(\xi_t)d\xi_t - r_t c - q_t e\right] \qquad \ldots(44)$$

subject to the original constraints (24)-(27). Forming the Lagrangean (28) (where S is now defined by (44)) and differentiating with respect to q_t and r_t we get the new optimality conditions

$$(1+r)^{-t}(x_t-e)-\lambda+v_t+\sum_{i=t+1}^{\infty}\rho_i = 0 \qquad \ldots(45a)$$

and

$$(1+r)^{-t}(x_t-c)+\mu_t-\rho_t = 0. \qquad \ldots(45b)$$

We now proceed to the analysis of the cases, $c>e$, $c=e$, and $c<e$.

(i) $c>e$

The analysis of this situation, where recycling costs exceed extraction costs, is a natural extension of the analysis in Section 5 where $e = 0$. As in Section 5, the analytic structure produces a three-stage consumption pattern. In Stage I, all of the consumption comes from unrecycled resource ($q_t>0$) and there is no recycling ($r_t = 0$). The difference between the rental and the marginal cost of extraction rises at the rate r—

$$x_t-e = (x_0-e)(1+r)^t \qquad \ldots(46)$$

—until the rental reaches the recycling cost, at which point the supply of fresh resource is exhausted.[1] Facing such a sequence of rentals, suppliers are indifferent among the options of renting the unused resource in any period through the cutoff period T.

Once the new resource has been exhausted, recycling begins and we are in Stage II. The rental is equal to the recycling cost in Stage II. When the demand at rental c exceeds the recycleable supply Q, the rental charge will rise above c, and we are in Stage III. As before, it is possible to move back and forth between Stages II and III (if demand fluctuates), but it is impossible to return to Stage I. Thus, if $c>e$, Figures 4 and 5 will still apply, with one change in Stage I. Now x_t-e, not x_t, rises geometrically until x_t reaches c.

The proof that this allocation is efficient is so similar to the proof for the case $e = 0$ that it is not given here. By following the steps of that proof with x_t-e substituted for x_t where appropriate, the optimality of the consumption stream stated above becomes evident.

Once again, it is worth noting that the value, or selling price, of a unit of the resource in its natural state is given by the shadow price of the supply constraint (24). In this case,

$$\lambda = (x_0-e)+\sum_{i=T}^{\infty}\rho_i. \qquad \ldots(47)$$

[1] As before, the rental in the period when the resource is exhausted may be exactly c or may lie between $c/(1+r)$ and c.

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This is interpreted as the total discounted surplus accruing to the owner of a unit of the resource at time 0. The first term is the discounted profit from extracting and renting the resource used up through period T; the second term is the discounted sum of the " stage III surplus ", where the surplus in period $t \geqq T$ is given by $\rho_t(1+r)^t$ as before. Again, no surplus accrues during Stage II, when the good rents at marginal cost c.

The selling price in period t is given by

$$\pi_t = \begin{cases} x_0 - e + \sum_{i=T}^{\infty} \rho_i & \text{for } t \leqq T, \\ \sum_{i=t}^{\infty} \rho_i & \text{for } t > T. \end{cases}$$

(ii) $c = e$

The case where extraction cost equals recycling cost is trivial. As long as the single-period demand at the rental c stays below Q, it does not much matter whether the resource is extracted or recycled. Thus, Stages I and II are effectively combined. In Stages I-II, the market rental will be equal to $c(=e)$ and Figure 5*a* applies. In Stage III, only recycling is possible since the resource is fully extracted. Here Figure 5*b* applies, and the rental will exceed the marginal cost c. The optimality of this allocation follows directly from (45). Set $c = e$, and follow the steps used in the proof for the model with zero extraction costs.

Since there is no surplus in Stage I in our present case, the selling price in period 0 is given by

$$\lambda = \sum_{i=0}^{\infty} \rho_i,$$

and the selling price in period t is given by

$$\pi_t = \sum_{i=t}^{\infty} \rho_i.$$

(iii) $c < e$

If the cost of recycling is *less* than the cost of extraction, the character of the market allocation changes. However, it does remain efficient, as will be demonstrated presently.

If extraction is more expensive than recycling, it will never pay to extract any more of the resource if demand at the rental c can be met by the previously extracted supply. Thus, if $d_t^{-1}(c) \leqq \sum_0^{t-1} q_i$, then $q_t = 0$. If there is demand in excess of the previously extracted supply (as must be the case, for example, in period 0), then more will be extracted.

Clearly the equilibrium rental in all periods for which the existing used supply meets demand at the rental c is, in fact, the recycling cost c. In all periods for which new extraction occurs, it must be the case that in equilibrium the present value of net return from extraction in all such periods are equal. Otherwise, suppliers would extract nothing in periods when this net return is less than in the period in which it reaches a maximum. The present value of net return from extracting one unit in period t is given by

$$v_t = (1+r)^{-t}(x_t - e) + \sum_{i=t+1}^{\infty} (1+r)^{-i}(x_i - c), \qquad \ldots(48)$$

and these v_t must be equal for all periods t in which extraction occurs. In some cases, suppliers will suffer an initial loss of $(e - x_t)$, to be made up in later periods when recycling occurs and the rental exceeds marginal cost.

As in the case $c = e$, Stages I and II become ill-defined. Prior to the period when all of the resource is finally extracted, there will be periods in which $q_t = 0$ and $r_t > 0$,

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and periods in which $q_t > 0$ and $r_t > 0$. Except for period 0, there are no periods in which $q_t > 0$ and $r_t = 0$ since it always pays to recycle whatever is available.

Stage III is also ill-defined in this case. It is possible that the rental will rise above marginal recycling cost even before all of the resource has been extracted. Thus, there may be Stage III-type profits prior to complete extraction. The higher the extraction cost, the more likely this is to occur.

The proof of the optimality of this result follows from (45) and another application of the Kuhn-Tucker conditions. Consider first the case $\mu_t = \nu_t = 0$ ($q_t > 0$, $r_t > 0$), a case which we expect to occur more often here than in the earlier analyses. In this case, (45*a*) and (45*b*) imply that

$$x_t = c + \rho_t (1+r)^t \qquad \ldots(49)$$

and

$$\lambda = (1+r)^{-t}(x_t - e) + \sum_{i=t+1}^{\infty} \rho_i. \qquad \ldots(50)$$

Substituting for ρ_t from (49) into (50) we get

$$\lambda = (1+r)^{-t}(x_t - e) + \sum_{i=t+1}^{\infty} (1+r)^{-i}(x_i - c). \qquad \ldots(51)$$

In the case $\mu_t = 0$, $\nu_t > 0$ ($q_t = 0$, $r_t > 0$), (45*b*) gives (49) as well. In the case $\mu_t > 0$, $\nu_t = 0$ ($q_t > 0$, $r_t = 0$), which, it was argued, cannot occur except in period 0, we see from (45*a*) and (45*b*) that

$$\lambda = (x_0 - e) + \sum_{i=1}^{\infty} \rho_i.$$

Substituting for ρ_i from (49) this becomes

$$\lambda = (x_0 - e) + \sum_{i=1}^{\infty} (1+r)^{-i}(x_i - c). \qquad \ldots(52)$$

Together (51) and (52) give the market equilibrium result (48). Thus the market allocation is again seen to be efficient. The price, or value, of a unit of the resource in period 0 is again represented by the shadow price λ, which is given by (52).

7. RECYCLEABLE RESOURCES WITH INCREASING MARGINAL EXTRACTION COSTS

The most general recycling model to be considered specifies that the marginal cost of extraction is not constant, but is increasing as the amount extracted increases. Let $e(q)$ be the total cost of extracting the first q units, independent of the period in which extraction occurs.

It is useful to define a notion of marginal cost which includes the increments to present and future cost. Consider a sequence of quantities extracted $\{q_t\}$. Then the total present-value cost of extraction is given by

$$E = e(q_0) + \sum_{t=1}^{\infty} \left[e\left(\sum_{i=0}^{t} q_i\right) - e\left(\sum_{i=0}^{t-1} q_i\right)\right](1+r)^{-t}. \qquad \ldots(53)$$

The *effective marginal cost* of extracting an additional bit in period 0 is given by

$$e_0 = e'(q_0) + \sum_{k=1}^{\infty} \left[e'\left(\sum_{i=0}^{k} q_i\right) - e'\left(\sum_{i=0}^{k-1} q_i\right)\right](1+r)^{-k},$$

and, in general, the effective marginal cost of extraction in period t is given by

$$e_t = e'\left(\sum_{i=0}^{t} q_i\right) + \sum_{k=t+1}^{\infty} \left[e'\left(\sum_{i=0}^{k} q_i\right) - e'\left(\sum_{i=0}^{k-1} q_i\right)\right](1+r)^{t-k}. \qquad \ldots(54)$$

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It was demonstrated in [14] that the market equilibrium (and also the efficient) allocation is such that price minus effective marginal cost grows at the rate r. The variable effective marginal cost plays the same role in the analysis as a constant marginal cost.

In the model with recycling, the analysis for the cases of constant marginal cost of extraction e applies directly to the case of variable effective marginal cost e_t. As long as the marginal cost e_t is less than the cost of recycling, rental less marginal cost rises at the rate r:

$$(x_t - e_t) = (x_0 - e_0)(1+r)^t. \qquad \ldots(55)$$

This trend continues until the marginal cost e_t reaches the recycling cost c. Then, the behaviour of the model with constant extraction cost $e > c$ takes over. Recycling and extraction will occur in tandem, and the rental in periods where extraction occurs will satisfy

$$(1+r)^{-t}(x_t - e_t) + \sum_{i=t+1}^{\infty} (1+r)^{-i}(x_i - c) = \lambda = \text{constant} \qquad \ldots(56)$$

for all such t. The shadow price λ once again represents the market value of a unit of the unextracted resource.

Empirical experience with a variety of resources bears out the results of this model. At first it does not pay to recycle, and the resource is consumed such that price minus effective marginal cost of extraction grows at the rate r. Foresighted suppliers may choose to lease the commodity, storing the spent resource until recycling becomes worthwhile. Alternatively if they sell, they will add on to the base price an increment which represents the increase in discounted surplus (rental less recyling cost) in all successive periods.[1] Eventually, expanding demand will make it pay to recycle as well as extract.[2] Extraction now will be occurring in periods of high demand. The rental will fluctuate at levels above the recycling cost. The sale price will increasingly include the recycling value of the resource, as this value comes to dominate recycling (and storage) costs. If the resource is ever fully extracted—no materials appear to have reached this state at present—demand will be met entirely by recycling.

8. MODELS OF REPRODUCIBLE RESOURCES

Our earlier paper [14] and the analysis given here cover two of the three major categories of resources in limited supply. The third category includes resources that reproduce themselves: rabbits, fish, micro-organisms, and perhaps best known to economists, the stock of capital. It also includes resources whose supply can be increased at some fixed rates, and resources that can be produced through normal economic processes. The application of energy from the sun or other natural forces represent fixed-rate processes. Similarly, the growing of timber in some restricted area is represented by the model of fixed-rate regeneration. The normal economic processes case is perhaps the most common. It is structurally equivalent to the model of Section 7, where the marginal cost of production plays a role analogous to the effective marginal cost of extraction in the Section 5 model.

Our survey of these reproducible resource models, many of which are well known, has revealed no phenomena that run counter to the intutition developed in this paper, although it has revealed a number that are external to this analysis. For example, the self-reproducing models generate intriguing questions relating to the optimal length of the investment period, and to inherent externalities in growth processes that are non-linear.

1 The magnitude of this increment will be given by $(1+r)^t \Sigma_t^\infty \rho_i$ if the good is sold in period t.
2 If demand is contracting, the sales price will be diminishing and no new extraction will be taking place.

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9. CONCLUSION

In [14] it was shown that free market behaviour, under certain conditions, will allocate a fixed-supply, strictly depletable resource in an intertemporally efficient manner. This paper complements that analysis in two ways.

First, it examines the behaviour of markets with depletable resources where market interest rates are determined endogenously. Second, it develops several models of markets for recycleable resources. The most general case has extraction costs increase with the amount extracted. After an initial period of extraction only, there will be a period of coexistence of recycling and extraction activities. This model provides some insight into present recycling activities, and some that are projected to become of consequence in the not-too-distant future.

We recognize, as a number of critics have pointed out, that neither capital nor contingent claims markets in the real world are as perfect as those we postulate here. (To the best of our knowledge there is no measure of degree of imperfection; such would be useful for this type of analysis.) But this does not imply that in the present second-best situation either (1) that the imperfect market processes lead to too rapid a rate of resource consumption, or (2) that government participation in these markets would lead to an improved situation.

In the recycleable resource context, for example, the federal government is at this very moment said to be considering a drastic revision in its policies of timber-cutting in our national forests, not in the direction conservationists would prefer. Any new decision, or indeed its predecessors, may be motivated by temporary political considerations or transient economic phenomena much more than by any long-run strategy on effective resource use.

The argument should at least be entertained that long-run economic forces, even if they are expressed imperfectly, may be a better guide to action than the decisions of legislators and bureaucrats.[1] If biases in the market are both identifiable and capable of estimation, then we can provide appropriate compensation mechanisms to encourage or discourage use. The much, perhaps justifiably, maligned depletion allowances are an example of such " market-correcting " tools in action.

Hopefully the findings of this paper will lead the proponents of resource preservation measures to focus their attentions on identifying divergences between the real world and the perfect world we have analysed. By directing their search toward such traditional issues as externalities and market imperfections associated with depletable and recycleable resources, they may discover valid new arguments for the policies they propose.

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[1] That a corporate management survives but a few years does not sway our thinking on the too-high-resource-use argument. Corporate officers pay attention to stock prices as well as earnings, and the former reflect expected future profits to a very significant degree. When a management departs, the company survives, and stockholders and potential stockholders know that.

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