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Author(s): Richard J. Zeckhauser

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Puzzles and Problems

EDITOR'S NOTE: This section of the *Journal* arises out of the need for a new means of communicating brief solutions to interesting problems as they develop in everyday research or in the classroom. Readers are encouraged to submit problems and/or solutions bearing on theoretical analysis as well as surprising and unusual economic facts or correlations. "Puzzles and Problems" is edited by Arnold Zellner, and communications should be sent to him at the Graduate School of Business, University of Chicago, Chicago, Illinois 60637.

A Tale of Probable Regions: A Statistical Fable

Richard J. Zeckhauser

Harvard University

It was the king's birthday and as usual it was to be celebrated with feats of skill and bravery. The final event was the BATTLE TO THE DEATH which brought the fiercest warrior in the Southern Barony into mortal combat against his northern counterpart. The battle this year was especially significant, for the victorious barony was to become the home of the new Royal University.

The warriors parried and jousted with such fury that soon all identifying badges had been torn from them or rendered useless. The battle took its toll and in time both men lay dying. The judges rushed to the dying men to determine which one would expire last. As one warrior breathed his last, the other mumbled his final words, "I am Fitzgarth."

This fable has been circulated as RAND Corporation P-2951.

This was a grave crisis as it was not known whence hailed Fitzgarth. Fortunately, the king was both wise and learned. He summoned his two sons, the rulers of the baronies, to his court and announced that the outcome of the battle would be determined on a statistical basis. The distribution of names in the two baronies had been tabulated in the recent royal census (table 1). It merely remained to make a statistical calculation as to Fitzgarth's more probable origin. Each barony appointed its official statistician; the north picked a soul by the name of Fisher and the south chose a chap by the name of Bayes. The royal census gave the breakdown of names in the two baronies as shown in table 1.

The next morning the royal court was summoned. Bayes spoke first, representing the Southern Barony:

Your Royal Highness—I beg the indulgence of the court to present the case for my belief that a priori it was with probability .9 that the victorious warrior would be a representative of the Southern Barony. In the south our warrior tradition leads us to begin training our male youths for combat at the age of six, four years before their northern counterparts commence training. In past contests, southern warriors have been victorious . . .

The king interrupted sharply,

Enough of this hanky panky. We care not about past results or training methods. This was a fair contest; we had never seen these two warriors in combat previously. Go back and recalculate your results without any attempts at sophistry.

Bayes appeared crestfallen, but he responded quickly,

Your Royal Highness—employing my methods of statistical evaluation, it is not necessary to go through tedious calculations for this problem. You have asked me to work on the basis that prior to learning the name of the victorious warrior it was equally likely that he had come from the Northern or Southern Barony. Given this belief—what I would call a prior of ($\frac{1}{2}$, $\frac{1}{2}$)—I would have to conclude that it was more likely that Fitzgarth represented the Northern Barony. In more exact terms, it's a probable nine to seven.¹

The king once again took the floor,

I was looking for a bit more of a fight, but I guess that fate was against the south. For purposes of formality let us have the analysis of the northern statistician.

¹ The nine to seven odds ratio quoted by Bayes represents the ratio of forty-five northern Fitzgarths to thirty-five southern Fitzgarths, each weighted by its prior probability, one-half.

TABLE 1
DISTRIBUTION OF NAMES—ROYAL CENSUS

Name and Location	Number	Name and Location	Number
Templeton:		Carver:	
North.....	120	North.....	0
South.....	20	South.....	25
Phipps-Warner:		Callison:	
North.....	105	North.....	0
South.....	25	South.....	25
Lancelot:		Horfath:	
North.....	95	North.....	0
South.....	25	South.....	25
Carembert:		Garvey:	
North.....	90	North.....	0
South.....	25	South.....	25
Runciman:		Lickland:	
North.....	85	North.....	0
South.....	25	South.....	25
Halifax:		Portsmouth:	
North.....	80	North.....	0
South.....	25	South.....	25
Norfolk:		Sherman:	
North.....	75	North.....	0
South.....	25	South.....	25
Castain:		Garrison:	
North.....	70	North.....	0
South.....	25	South.....	25
Lindsey:		Sterbond:	
North.....	65	North.....	0
South.....	25	South.....	25
Smythe:		Mott-Smith:	
North.....	60	North.....	0
South.....	25	South.....	25
Fisher:		Cravish:	
North.....	55	North.....	0
South.....	25	South.....	25
Bayes:		Archer:	
North.....	50	North.....	0
South.....	5	South.....	25
Fitzgarth:		Carleton:	
North.....	45	North.....	0
South.....	35	South.....	25
Partridge:		Crawford:	
North.....	5	North.....	0
South.....	15	South.....	25
Tavistock:		Dumbart:	
North.....	0	North.....	0
South.....	25	South.....	25
Strickfin:		Craigmyle:	
North.....	0	North.....	0
South.....	25	South.....	25
Grosvenor:		Oliphant:	
North.....	0	North.....	0
South.....	25	South.....	25
Jessup:		Street:	
North.....	0	North.....	0
South.....	25	South.....	25
Dougherty:		Masser:	
North.....	0	North.....	0
South.....	25	South.....	25
Trenttynne:		Cartwright:	
North.....	0	North.....	0
South.....	25	South.....	25
Tennyson:			
North.....	0	Total north.....	1,000
South.....	25	Total south.....	1,000

Fisher had looked dejected throughout the proceedings. It was difficult to understand why he should be so downcast when his barony was soon to be proclaimed the home of the Royal University. Fisher spoke quaveringly,

Your Royal Highness—unfortunately, I cannot claim the victory for my barony that the southern statistician has recently conceded. In conducting my analysis, I have dutifully followed the statistical testing procedures that have been developed at Rothamsted, the Royal Agricultural Station which graces our barony.

I tested the null hypothesis that Fitzgarth was from the Northern Barony against the alternative hypothesis that he was from the Southern Barony. The information that I had to work with in this instance was the name of the victorious warrior. To be perfectly objective, I considered not only his actual name, but all other names he might have had. Some of these, such as Templeton, would give strong evidence that the victor had represented the north. Many others would be names known only to be found in the south. In my test I employed data on the incidence of names in the two baronies.

Now I wanted only a .05 chance of rejecting the null hypothesis when in fact it was valid. Following the now-classic Rothamsted procedures, I picked as my critical region those names which together had a .05 chance of occurring in the north which also had the greatest chance of occurring in the south. This region contains the names Partridge and Fitzgarth. Alas, the observation is the name Fitzgarth; it lies within this critical region. I was forced to reject the null hypothesis as the results were not significant at the .05 level. This outcome was unfortunate. To give my barony every benefit of the doubt. . . .

The king interrupted once again,

I imagine that “benefit of the doubt” is a statistical euphemism for something somewhat more insidious. Continue.

Fisher went on,

Yes, Your Majesty. In the hope that I could also reject the hypothesis that Fitzgarth was from the south, and thus save the day for the north,² I constructed a new test. This time I established the null hypothesis that Fitzgarth was from the

² In the event that both the Northern and Southern Baronies would be rejected as the null hypotheses for Fitzgarth's origin (a possibility using classical methods), the king would declare Fisher's analysis to be ruled irrelevant and he would award the north the University on the basis of the Bayes analysis.

Southern Barony against the alternative hypothesis that he was from the Northern Barony. Unfortunately, I was unable to reject the null hypothesis in this instance.³ I am afraid that I have failed my home, the Northern Barony, but the truth must win out.

The king was furious at this display of cowardice on the part of the statisticians. He had them thrown into the dungeon for life in the hope that they might learn of bravery and perhaps intellectual honesty.

In the meanwhile, the kingdom was in turmoil as the grave question as to the location of the Royal University was still undecided. Late that night, the official court sage came to the king and told him that two well-known statisticians had been living in the kingdom by the names of R. A. Fisher and Thomas Bayes. The king demanded that they be brought to him immediately. What is the probability that they had to disturb the jailer?

³ It was discovered later that Fisher had initially tested this hypothesis at the .05 level of significance. He was not able to reject the null hypothesis at this level, so he retested it using an unrestrictive .25 significance level. Unfortunately for him, he still was unable to reject the null hypothesis. (The best .25 critical region is Bayes, Templeton, Phipps-Warner, Lancelot, Carembert, Runciman, Halifax, Norfolk, Castain, Lindsey, and Smythe.)