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Selling Procedures with Private Information and Common Values

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The seller posted-price procedure is probably the most common method for making transactions in modern economies. We analyze the performance of posted pricing for transactions having significant common-value elements. In a model of two-sided private information, we characterize the fully revealing, perfect equilibrium offer strategy of the seller. We also characterize equilibrium behavior under two other pricing procedures—a sealed-bid procedure and a direct revelation mechanism. Finally, we examine the efficiency of these procedures and show that as the degree of common values increases, fewer mutually beneficial agreements are attained.
(*Bargaining; Common Values; Game Theory; Asymmetric Information*)

"Low prices informing, buyers take warning. High prices set right, common values delight."

Old Seller's Rime

1. Introduction

In many commonly encountered bargaining situations, each agent possesses private information that bears on the potential value of the transaction. Examples range from a buyer and seller negotiating the sale of an item to a pair of disputants attempting to settle their conflict through negotiation rather than adjudication. Typically, bargaining proceeds by exchanges of offers. The present analysis highlights two elements of these situations.

First, the payoffs from such agreements almost always involve common values as well as private values. For instance, the long-term worth of a target firm if taken over by a would-be acquirer has a private-value element (depending on the acquirer's management capabilities) and a substantial common-value element (based on existing products and market conditions). Similarly, in a joint venture, the respective profits of the partners depend both on their private capabilities and circumstances and on common factors such as market conditions. Finally, the potential litigation value of a case has a large common-value element (the expected value of the court award) as well as private-value elements (reputation concerns and private legal costs).

Second, the exchange of offers, besides suggesting potential terms of agreements, also transmits information concerning these private- and common-value elements. For instance, when the transaction contains a significant common-value element, a seller's offer will convey information not only about his own value, but also about the potential value to the buyer. The buyer must combine such inferences with his other information to determine her expected value. (Throughout we refer to the seller as he, the buyer as she.) One objective of this paper is to characterize equilibrium offer and acceptance behavior taking these subtle inferences into account. A second objective is to examine the joint effect of different degrees of common values and alternative offer procedures on the efficiency of bargaining.

As is well known, under private information, strategic bargaining behavior leads to inefficient outcomes in equilibrium. However, almost all the research to date has considered models in which the bargainers hold *in-*

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dependent private values.[1] Notable exceptions include Gresik's (1991) analysis of optimal trading mechanisms with dependent values and Vincent's (1989) and Evans' (1989) studies of sequential bargaining with common and correlated values. The neglect of dependent-value models by the bargaining literature is curious. (The auction literature, by contrast, has investigated many dependent-value models and has not been tied to the independent-value assumption.) The present paper extends the analysis of bargaining behavior and performance to settings exhibiting value dependence.

Section 2 presents a bargaining model in which the players have private signals bearing on the potential value of the transaction. The information structure allows for varying degrees of private and common values. We examine the performance of the simplest and most commonly used pricing procedure: the seller names a price which the buyer can then accept or reject. The analysis characterizes the perfect Bayesian equilibrium associated with the seller-offer procedure. Attention is paid not only to the inferences the buyer should draw from seller offers, but also to the inferences the seller must make in advance from the fact of buyer acceptance. (Acceptance implies that the item is worth more than the seller expected.) This section also considers a second, symmetric procedure in which both agents submit simultaneous offers, and the final price splits the difference between them.

Section 3 presents a basic result concerning inefficiency in bargaining. Using the direct revelation approach, the analysis shows that increasing the common-value element in the players' reservation prices reduces bargaining efficiency. The incidence of bargaining impasses, well known in independent-value models, is exacerbated by the introduction of dependent values. This result holds for any bargaining procedure. In fact, the general analysis underscores a main drawback of the seller-offer procedure—that it reveals *too much* information. The very inferences that the buyer draws from seller offers limit the transactions that a rational buyer can accept. Section 4 offers a brief summary and concluding remarks.

[1] Early work includes Fudenberg and Tirole (1983), Myerson and Satterthwaite (1983), and Chatterjee and Samuelson (1983). For more recent surveys of this bargaining literature, see Linhart (1989) and Kennan and Wilson (1993).

2. Offer Procedures

A buyer and a seller (both risk neutral) are attempting to negotiate the sale of an indivisible good. Each player has private information in the form of a signal bearing on the potential value of the good. Let x denote the seller's private signal and y the buyer's private signal. We make two assumptions about player signals and values.

ASSUMPTION 1. *The signals are drawn independently from distributions $F(x)$ with support $[0, 1]$, and $G(y)$ with support $[\underline{y}, \bar{y}]$.*

ASSUMPTION 2. *Each player's value for the good is a separable function of x and y. In particular, the seller's value is $v_s = x + \alpha y$ and the buyer's value is $v_b = y + \beta x$, where α and β are in the unit interval. In addition, these values are independent of income effects.*

REMARK 1. The separability requirement in Assumption 2 implies that the effect of a change in a player's private signal on v_s or v_b is independent of the other's (unknown) signal. As we shall see, this assumption greatly simplifies each player's equilibrium behavior. The independence of Assumption 1 serves a similar simplifying purpose. If dependence were the rule, each player would have to draw a different probabilistic inference about the other's signal, depending on his own signal. By adopting Assumption 2, we abstract from this interesting but complex problem.

REMARK 2. In related work, Gresik (1991) considers a more general value structure, assuming only that any change in a player's private signal will change his own valuation for the item more than it will change his partner's. Our assumption that α and β are smaller than one ensures that this is the case. Note that if α and β are both zero, the players' values are purely private and independent. If α and β are both one, the item has a common value for the players (though neither knows this value absent the other's information). Thus, by varying these parameters, one can introduce a greater or lesser common-value element into the transaction. A second important restriction in Assumption 2 is that the players' values are separable, *linear* functions of the signals.

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2.1. Seller-Offer Procedure

Our main focus is on the traditional bargaining procedure whereby the seller makes a single price offer, which the buyer can accept or reject. (The analysis if the buyer makes the offer is completely analogous.) Let p denote a particular price offer and $p = P(x)$ denote the seller's offer function, i.e., the schedule listing the seller's offer for each possible value of his private signal. In turn, the buyer's acceptance behavior can be summarized by a "cut-off" value r. The buyer will accept the seller's offer if and only if $y \geq r$ (i.e., only if her private signal is greater than or equal to this cut-off value). The buyer's complete acceptance strategy can be summarized by the function $r = R^*(p)$, which sets a cut-off value given each offer of the seller.

For the functions $P(\ )$ and $R^*(\ )$ to constitute a Nash equilibrium, it must be the case that each player maximizes his expected profit given the strategy of the other. We restrict our attention to equilibria in which $P(\ )$ is differentiable and strictly increasing—that is, the seller's offer strategy is fully revealing. In this case, the buyer infers x from the seller's offer p. Let $P^{-1}(\ )$ denote the seller's inverse offer function. The buyer infers x from p according to $x = P^{-1}(p)$. Her optimal acceptance strategy is to purchase the item if and only if:

$$\pi_b = v_b - p = (y + \beta x) - p \geq 0,$$

or equivalently:

$$y \geq r \equiv p - \beta P^{-1}(p). \tag{1}$$

Thus, Equation (1) gives the buyer's optimal cut-off value. This acceptance strategy applies for quoted prices in the range of the seller's offer strategy. Let $\underline{p}$ and $\bar{p}$ denote the smallest and largest equilibrium seller offers respectively. To describe a *perfect* equilibrium, we need to specify the buyer's behavior in the event of a seller quote *outside* the interval $[\underline{p}, \bar{p}]$. To cover this out-of-equilibrium event, we impose two reasonable conditions on the buyer's beliefs: (1) If $p \geq \bar{p}$, then the buyer concludes that $x = 1$ and adopts the cut-off value, $r = p - \beta$; (2) If $p \leq \underline{p}$, then the buyer concludes that $x = 0$ and adopts the cut-off value, $r = p$. As we will show, the second condition is sufficient to characterize the seller's differentiable offer strategy.

We now characterize the seller's equilibrium strategy. When he holds signal x, the seller's expected profit is:

$$\pi_s(x) = \int_r^{\bar{y}} [p - v_s]g(y)dy = \int_r^{\bar{y}} [p - (x + \alpha y)]g(y)dy, \tag{2}$$

The seller's task is to choose p to maximize $\pi_s(x)$ subject to the buyer's acceptance strategy, i.e., r defined in (1). Instead of optimizing with respect to p, it is more convenient to think of the seller as reporting his value x, then applying the offer function $P(\ )$. To be specific, we can write the seller's offer as $P(z)$ and view him as choosing z. Then $P(\ )$ is an optimal strategy if and only if he can do no better than choose $z = x$ for all x. If the seller chooses z, then the buyer infers this to be the value of x. That is, she hears the offer $P(z)$, inverts this function to obtain the value of x (in equilibrium), and establishes the cut-off value $r = p - \beta z$. This enables us to rewrite the seller's expected profit as:

$$\begin{aligned}\pi_s(x, z) &= \int_{R(z)}^{\bar{y}} [P(z) - x - \alpha y]g(y)dy \\ &= \int_{R(z)}^{\bar{y}} [R(z) + \beta z - x - \alpha y]g(y)dy. \end{aligned}\tag{3}$$

Here, $r = R(z)$ denotes the buyer's cut-off value as a function of z. (We have also used the fact that $r = p - \beta z$ or, equivalently, $p = r + \beta z$.) The partial derivative of $\pi_s(x, z)$ with respect to z is:

$$\begin{aligned}\partial\pi_s/\partial z = &\int_{R(z)}^{\bar{y}} (R'(z) + \beta)g(y)dy \\ &- R'(z)[R(z) + \beta z - x - \alpha R(z)]g(R(z)), \end{aligned}\tag{4}$$

where R' denotes dR/dz. In equilibrium, $\partial\pi/\partial z = 0$ at $z = x$ for all x. After some rearrangement, we arrive at the differential equation:

$$\begin{aligned}R'\{[(1 - \beta)x - (1 - \alpha)R]g(R) + (1 - G(R)\} \\ + \beta(1 - G(R)) = 0. \end{aligned}\tag{5}$$

In equilibrium, $R'(\ )$ must be nonnegative.[2] This means that an increase in x (accompanied by an increase

[2] To see this, let $W(x, z)$ denote the right-hand side of (4). In equilibrium, $W(x, x) = 0$ for all x or equivalently, $W(z, z) = 0$ for all z. Subtract $W(z, z)$ from (4) to cancel out all terms but the single one involving x. We find: $W(x, z) \equiv \partial\pi_s/\partial z = R'(z)(x - z)g(R(z))$. For $z = x$ to be optimal, $W(x, z)$ must be nonnegative for $z < x$ and nonpositive for $z > x$. Thus, $R'(x)$ must be nonnegative.

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in p) reduces the probability of an acceptance by the buyer. Because $P(x) = R(x) + \beta x$, it follows that $P'(x) = R'(x) + \beta$. In equilibrium, the seller's marginal price increase as x increases is at least as great as the buyer's marginal benefit. Thus, a higher price is always disadvantageous to the buyer. We summarize these findings in the following proposition.

PROPOSITION 1. *In a separating equilibrium, the seller's optimal offer strategy is given by* $P(x) = R(x) + \beta x$, *where* $R'(x)$ *is nonnegative and satisfies differential Equation* (5).

A Uniform Example. For arbitrary distribution functions, the differential equation in (5) has no general, closed-form solution. Furthermore, considerable care must be taken to establish appropriate boundary conditions. For special cases, however, complete solutions are available, and these are instructive. As an example, suppose that the buyer's signal y is uniformly distributed in the unit interval—that is, $G(R) = R$ and $g(R) = 1$. Employing the changes of variables, $t = T(x) = R(x) - 1$ and $s = x - (1 - \alpha)/(1 - \beta)$, we can rewrite (5) as:

$$T'[(1 - \beta)s - (2 - \alpha)T] - \beta T = 0.$$

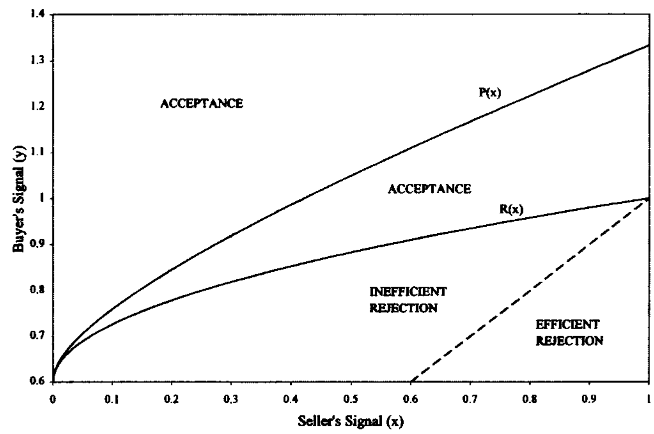
This differential equation has the closed-form solution:

$$s = k|t|^{(1-\beta)/\beta} + [(2 - \alpha)/(1 - 2\beta)]t, \qquad (6)$$

where k is a constant of integration (and $\beta \neq 0$ or 0.5). From this family of solutions (parametrized by k), the unique, perfect-equilibrium offer strategy can be identified by invoking the out-of-equilibrium conditions. The essence of the argument is that the seller should not be able to profit by making an offer below the range of equilibrium offers. As shown in Appendix A, this requirement implies the boundary condition $r = 1/(2 - \alpha)$ at $x = 0$. In turn, this boundary condition can be substituted into (6) to compute k.

To illustrate the nature of the equilibrium, Figure 1 shows the seller's optimal offer function for the case, $\alpha = \beta = \frac{1}{3}$. The graph also shows the buyer's acceptance region. (Remember that the buyer deduces x from $P(x)$ and establishes the cut-off value $r = P - \beta x$.) Finally, the area above the dashed line indicates the joint realizations of signals supporting mutually beneficial agreements, i.e., $v_b \geq v_s$. As the figure shows, the seller uses a markup strategy—that is, he sets his price above his expected personal value of the item, conditional on his offer being accepted. Thus, the acceptance region is smaller than the region of mutually beneficial agreements. Strategic bargaining often leads to disagreements even when gains from trade are available.

Figure 1



By inspecting the expressions for $P(x)$ and $R(x)$, we can identify a number of comparative statics results for the uniform case. First, the seller shifts to uniformly higher offers with an increase in α or β. The intuitive explanation is straightforward. If the seller's expected value for the item increases (via α), he naturally asks for higher prices. Similarly, a savvy seller should raise his prices if there is an increase in the potential value to the other side (β). Next, it is natural to ask: What effect do changes in the bargaining environment have on the realized gains from trade? For the uniform case, we can derive closed-form expressions for the players' expected profits. For $\beta \leq \alpha$, these are:

$$\pi_s = \int_0^1 \pi_s(x)dx = \frac{1}{6}\frac{(1 - \alpha)^3(2 + 3\beta)}{(2 - \alpha)(2 - \beta)(1 + \beta)}, \quad \text{and}$$

$$\pi_b = \int_0^1 \pi_b(y)dy = \frac{1}{6}\frac{(1 - \alpha)^3}{(2 - \alpha)^2(1 + \beta)}.$$

Note that

$$\pi_s = [(2 + 3\beta)(2 - \alpha)/(2 - \beta)]\pi_b.$$

Hence, the seller's ex ante expected profit is always strictly greater than the buyer's. Efficiency under the seller-offer procedure can be judged by comparing total

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realized expected profits to the potential expected trading gains, $E[v_b - v_s | v_b \geq v_s]$. In the uniform case, the potential trading gains come to: $\pi^* = (1/6)(1 - \alpha)^2 / (1 - \beta)$. Comparing $\pi_s + \pi_b$ to π^*, we find that the ratio of realized trading gains to potential gains declines as α and/or β increases.

Summary. Though the precise formulas for the seller and buyer strategies are somewhat complex, the qualitative characteristics of the equilibrium are straightforward. We have seen that the seller-offer procedure yields a fully separating equilibrium. The buyer recognizes that a higher price quote implies an item of greater value (both to the buyer and the seller) and sets her acceptance strategy accordingly.

Split-the-Difference Procedure

We can gain additional insight into the strategic elements associated with common values by examining a symmetric offer procedure. Consider a particular one-shot negotiation method (following Chatterjee and Samuelson 1983):

Under split-the-difference (SD) offers, the seller and the buyer submit sealed offers denoted respectively by s and b. If the buyer's offer is greater or equal to the seller's demand ($b \geq s$), then a negotiated agreement is concluded at the terms $P = (b + s)/2$. If s exceeds b, there is no agreement, and bargaining terminates.

There are two key differences between this procedure and the seller-offer procedure. First, the SD procedure solicits offers from both sides. Since the agents' offers depend on their respective private signals, so does the final price. By contrast, the final price when the seller makes the only offer necessarily reflects only his private signal. In this sense, the final price incorporates more information under the SD procedure than under the seller-offer procedure. Second, the inferences the buyer draws under the two methods are quite different. When the seller makes the only offer, the buyer infers perfectly the seller's signal. By contrast, under the SD method, the buyer does not know the seller's offer (or signal) at the time she makes her own offer. Instead, she must ask herself: What inference can I draw about the seller's signal (and, therefore, the value of the item) conditional on an agreement taking place, $s \leq b$? In this sense, the SD procedure conveys less information to the buyer than the seller-offer procedure.

In contrast to the seller-offer procedure, there are a multitude of (perfect) equilibria in the sealed-offer procedure.[3] Consider once again the example of uniformly distributed values, $F(x) = x$ and $G(y) = y$. It is straightforward to check that linear strategies constitute an equilibrium.[4] (For arbitrary distributions, Appendix B derives the linked differential equations characterizing equilibria in differentiable strategies.) In addition, we restrict attention to the symmetric case in which $\alpha = \beta$ and let α denote this common value. Then, the equilibrium offer strategies are:

$$s = S(x) = s_0 + s_1 x, \quad b = B(x) = b_0 + b_1 y,$$

where $s_1 = b_1 = (2/3)(1 + \alpha)$, $s_0 = 0.5s_1/(2 - s_1)$, and $b_0 = 0.5s_1(1 - s_1)/(2 - s_1)$. Note that the seller's offer function is uniformly greater than the buyer's—$S(x) > B(x)$, for any value of x. Thus, some agreements are missed ($b < s$) even when trading gains exist, i.e., when $v_b - v_s = (1 - \alpha)(y - x) > 0$.

The offer strategies respond to the degree of common values present in the transaction. An increase in α causes an upward shift in the seller's demands as well as the buyer's offers. (After all, increasing α increases the value of the item to both agents.) However, the upward shift in seller demands exceeds the shift in buyer offers, causing a fall in the frequency of agreements. An agreement occurs if and only if $b \geq s$, or equivalently, $y \geq x + 0.5(1 + \alpha)/(2 - \alpha)$. As α increases, the region of missed trades (and the associated dead-weight losses) increase. As α approaches one, agreements disappear altogether.

Table 1 compares the ex ante efficiency of the seller-offer procedure and the SD procedure for the uniform case with $\alpha = \beta$. For each procedure, the table lists the ratio of the realized gains from trade to the maximum

[3] For the sealed-bid procedure with independent values, Linhart et al. (1989) show that there exists a continuum of equilibria involving differentiable strategies as well as a multitude of equilibria involving step function strategies. In terms of performance, these equilibria range from the second-best ex ante efficient outcome to the complete inefficiency of no trade.

[4] We focus on the *linear* equilibrium for two reasons. First, it is the simplest possible differentiable equilibrium. Indeed, laboratory experiments to date (see Radner and Schotter 1989)) have shown that subjects have a strong tendency to play linear strategies. Second, as noted below, it has relatively desirable efficiency properties.

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Table 1 Efficiency Ratios in the Uniform Case with $\alpha = \beta$

	Seller Offer			Split-the-Difference	
	$\pi_s + \pi_b$	π_s	π_b	$\pi_s + \pi_b$	$\pi_s = \pi_b$
$\alpha = \beta = 0$	0.750	0.500	0.250	0.844	0.422
0.1	0.673	0.469	0.204	0.797	0.399
0.2	0.593	0.428	0.165	0.741	0.370
0.3	0.509	0.378	0.130	0.673	0.336
0.4	0.422	0.322	0.100	0.593	0.297
0.5	0.323	0.259	0.074	0.500	0.250
0.6	0.245	0.194	0.051	0.394	0.197
0.7	0.160	0.128	0.031	0.276	0.138
0.8	0.083	0.068	0.015	0.156	0.078

possible gains from trade. We observe that the SD procedure is significantly more efficient than the seller-offer procedure. As noted earlier, the seller-offer procedure produces very unequal payoffs to the agents. The buyer (who is disadvantaged under the seller-offer procedure) unambiguously prefers the equal treatment of the symmetric SD procedure. For low enough α, the seller prefers to retain the power to make a take-it-or-leave-it offer. But for α greater than 0.54, the seller's preference switches to the SD method because it yields superior expected profits.[5]

For this particular example, increasing the common-value element of the players' values reduces ex ante efficiency under either procedure. For instance, according to Table 1, the SD procedure achieves over 84% of the potential trading gains when values are independent ($\alpha = 0$). But this percentage falls drastically, from about 60% to 16%, as α increases from 0.4 to 0.8. The pattern for the seller-offer procedure is analogous. Of course, as α approaches one, the maximum feasible gains, as a fraction of the good's value to either player, also approach zero, in which case the inefficiency matters less. The next section shows that this result holds not only for this example but in general. Regardless of the bargaining procedure and the particular distributions, efficiency declines as the degree of value dependence increases.

[5] The efficiency advantage of the SD procedure does not depend on the prescribed symmetry of the example. Corresponding calculations when the players have different estimate distributions (retaining the uniform assumption) confirm the SD procedure's efficiency advantage, provided asymmetries are not "too" pronounced. One such asymmetry occurs if the support of the buyer's estimate is much narrower than the support of the seller's. (That is, there is much less uncertainty about the buyer's estimate than about the seller's.) In this case, efficiency dictates that the final price be solely determined by the seller's (informed) offer rather than by both players' offers.

3. Direct Revelation Mechanisms

It is well known that an equilibrium of any game under incomplete information, no matter how complicated the strategies, can be recast as a direct revelation mechanism (DRM) in which each side reports its private information (truthfully), and these revelations directly determine equilibrium outcomes. We will apply the direct revelation approach to characterize possible negotiation outcomes. This approach has the advantage that it allows us to dispense with the exact description of the negotiation process. Rather, the approach encompasses *all possible* bargaining or negotiation methods. Use of the direct revelation approach was pioneered by Myerson and Satterthwaite (1983) for bilateral monopoly and by Gresik and Satterthwaite (1989) for mechanisms involving many traders. Recently, Gresik (1991) has extended the approach to trading mechanisms among agents having dependent values. The present analysis draws on and complements Gresik's results.

The negotiated outcomes of the direct revelation mechanism can be summarized by two functions that map realizations of the parties' signals into outcomes: (1) $p(x, y)$, the probability that the good is transferred from seller to buyer, and (2) $t(x, y)$, the monetary amount paid by the buyer to the seller (unconditional on whether the good is transferred). In the play of the DRM, each side's *report* of his or her signal determines the likelihood of an agreement, $p(x, y)$, and the amount paid, $t(x, y)$. (Of course, the DRM must be designed to give each player the incentive to report a *true* signal.)

Consider the expected payoff of the seller when he holds signal x and reports his signal as z. This is:

$$\pi_s(x, z) = \int_{\underline{y}}^{\bar{y}} \{t(z, y) - p(z, y)[x + \alpha y]\}g(y)dy. \quad (7)$$

This payoff is the expected (unconditional) transfer he receives minus the value of the good in the case of a

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sale. Note that the probability of a sale and the transfer payment depend on his *reported* signal. To implement an equilibrium outcome, the DRM must ensure that each player maximizes his expected payoff by reporting his true signal—that is, the DRM must be *incentive compatible* (IC). For the seller, the direct revelation mechanism must satisfy $\pi_s(x, x) \geq \pi_s(x, z)$ for all possible reports z and all x. For ease of notation, we denote the seller's expected profit conditional on x and given a truthful report by $\pi_s(x) \equiv \pi_s(x, x)$. Also define $P(x) = E_y[p(x, y)]$, where E_y denotes the expectation of y, over the distribution $G(y)$. Thus, $P(x)$ denotes the seller's assessed probability of an agreement (in equilibrium) conditional on x. According to standard results in the direct revelation literature (see Myerson and Satterthwaite 1983), the IC conditions imply:

$$P(x) \text{ is nonincreasing in } x. \tag{8}$$

Roughly speaking, the higher is the seller's signal, then the higher must be the trading price in equilibrium, implying a lower probability of an agreement. A second implication is that

$$d\pi_s/dx = -P(x), \tag{9}$$

almost everywhere. An informal way to derive equation 9 is to apply the envelope theorem to Equation (7):

$$d\pi_s/dx = \partial\pi_s/\partial x + (\partial\pi_s/\partial z)(dz/dx) = \partial\pi_s/\partial x,$$

since an optimal report implies $\partial\pi_s/\partial z = 0$. Thus, we confirm $d\pi_s/dx = -P(x)$ since z must be identically equal to x to satisfy incentive compatibility.

The analysis for the buyer is similar. The buyer's expected profit when she holds signal y and reports z is:

$$\pi_b(y, z) = \int_0^1 \{p(x, z)[y + \beta x] - t(x, z)\} f(x)dx. \tag{10}$$

Incentive compatibility implies that

$$Q(y) \text{ is nondecreasing in } y, \quad \text{and} \tag{11}$$

$$d\pi_b/dy = Q(y), \tag{12}$$

almost everywhere, where $Q(y) = E_x[p(x, y)]$. Note that a higher signal for the buyer implies a higher probability of agreement (by (11)) and a greater expected profit for the buyer (by (12)).

A second requirement of the mechanism is that it be *individually rational* (IR)—that is, the players' conditional payoffs, $\pi_s(x)$ and $\pi_b(y)$, must be nonnegative for all x and y. From (12) and (15), the seller's and buyer's profits are nonincreasing and nondecreasing, respectively. Thus, the relevant IR constraints become

$$\pi_s(1) \geq 0 \quad \text{and} \quad \pi_b(\underline{y}) \geq 0. \tag{13}$$

That is, the player must be satisfied with his expected payoff when he holds his least favorable signal (i.e., the greatest x for the seller or the lowest y for the buyer). In fact, since lump-sum transfers can be made between the parties, the equivalent IR constraint is simply

$$\pi_s(1) + \pi_b(\underline{y}) \geq 0. \tag{13'}$$

By integrating the seller and buyer IC constraints (see Appendix C), we can express this IR constraint in more usable form as:

$$\pi_s(1) + \pi_b(\underline{y}) = \int_{\underline{y}}^{\bar{y}} \int_0^1 \{[(1 - \alpha)y - (1 - G(y))/g(y)] - [(1 - \beta)x + F(x)/f(x)]\} p(x, y) f(x) g(y) dx dy \geq 0. \tag{14}$$

To sum up, implementing a bargaining outcome comes down to specifying functions $p(x, y)$ and $t(x, y)$ that satisfy the IC and IR conditions in (8), (9), (11), (12), and (14).[6]

Efficiency. Ex post efficiency requires that the good be sold if and only if the buyer's signal exceeds the seller's. That is, the bargaining mechanism must specify $p(x, y) = 1$ if and only if $v_b = y + \beta x \geq v_s = x + \alpha y$. It is well known (see Myerson and Satterthwaite 1983) that ex post efficiency is unobtainable when the bargainers hold private personal values. To verify this using the present framework, (1) set $\alpha = \beta = 0$ to invoke purely private values, and (2) confirm that setting $p(x, y) = 1$ for $y \geq x$ *violates* the IR constraint (14).

In an important article, Gresik (1991) has provided a general characterization of trading mechanisms show-

[6] Once the probability function of the DRM is established, the payment function can be established via Equations (7), (9), (10), and (12). Many different lump-sum transfers are possible, subject of course to the IR constraints in (13).

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ing that value dependence is also incompatible with ex post efficiency.[7] We demonstrate a complementary result—that bargaining efficiency declines as the degree of common values increases.

This effect can be captured by setting $\alpha = \beta$ and letting this common value increase from zero to one. Under this restriction, the maximum possible trading gains are:

$$E[v_b - v_s | v_b \geq v_s] = (1 - \alpha)E[y - x | y \geq x].$$

In turn, the realized (ex ante expected) trading gains for a particular mechanism are:

$$\pi_s + \pi_b = (1 - \alpha) \int_{\underline{y}}^{\bar{y}} \int_0^1 (y - x)p(x, y)f(x)g(y)dxdy.$$

The ratio of the players' realized trading gains to the maximum possible gains is a natural measure of bargaining performance. Note that this ratio does not depend on α. Now consider the problem of maximizing this ratio subject to the IR constraint in (14) as α increases from zero to one. As α increases, the only effect is on the IR constraint. Increasing α reduces the left side of the inequality in (14), thereby tightening the individual rationality constraint. In short, the ratio of realized trading gains to the maximum possible gains declines.[8] Thus, we have:

PROPOSITION 2. *Increasing the degree of common value between the reservation prices of the buyer and seller (increasing $\alpha = \beta$ on the interval 0 to 1) reduces the obtainable trading gains as a fraction of the maximum possible gain.*

Straightforward intuition underlies this result. When α and β are near zero, the magnitude of a player's profit depends mainly on attaining efficient agreements (to the extent afforded by the IC and IR constraints). However, as α and/or β increase, differences in reservation prices become much less important than the (nearly) common, *unknown* value the bargainers put on the transaction. Since each side knows only its own signal (half the relevant information), it has to be careful not to pay too much or accept too little for the transaction. Failing to draw the appropriate inferences, a player is likely to fall prey to the "winner's curse": buying an item at too dear a price or selling an item too cheaply. This strategic precaution creates a tighter IR constraint and implies fewer negotiated agreements.[9]

Maximizing the Gains from Trade. To maximize the expected trading gains subject to (14), we form the Lagrangian:

$$\pounds = \pi_s + \pi_b + \lambda(\pi_s(1) + \pi_b(\underline{y}))$$

$$= \int_{\underline{y}}^{\bar{y}} \int_0^1 H(x, y)p(x, y)f(x)g(y)dxdy, \quad \text{where}$$

$$H(x, y) \equiv (1 + \lambda)[(1 - \alpha)y - (1 - \beta)x] - \lambda[(1 - G(y))/g(y) + F(x)/f(x)].$$

Note that the Lagrangian is linear in $p(x, y)$. Therefore, trading gains are maximized by setting: $p(x, y) = 1$ for $H(x, y) \geq 0$ and $p(x, y) = 0$ for $H(x, y) < 0$.[10]

A Uniform Example. Again suppose $F(x) = x$ and $G(y) = y$ and let $\alpha = \beta$. It follows that

[7] Gresik's analysis allows a very general value structure, where each player's value is a function of all private signals. He shows that optimal trading mechanisms have deterministic solutions, provided any change in a player's private signal changes his (virtual) valuation more than it changes anyone else's. (This latter condition is assured by our assumption that α and β are smaller than one.) Thus, while we have focused on an additive structure for analytical simplicity, the approach extends to more general value structures.

[8] Alternatively, adopting the parameterizations $v_s = x/(1 - \alpha) + [\alpha/(1 - \alpha)]y$ and $v_b = y/(1 - \alpha) + [\alpha/(1 - \alpha)]x$ makes the same point. Here, the realized trading gains (as well as the maximum possible gains) are invariant with respect to α. Consequently, the maximand of $\pi_s + \pi_b$ declines as α increases (and the IR constraint tightens).

[9] As long as $\alpha = \beta < 1$, the individual rationality constraint allows "some" agreements, i.e., $p(x, y) = 1$ for sufficiently large y and small x. (To see this, note that the individual rationality constraint in (14) is nonbinding when $p(x, y) = 0$ for all x and y.) However, as $\alpha = \beta$ approaches one, the transaction becomes closer and closer to a case of pure common values. As is well known, in the pure common-value case ($\alpha = \beta = 1$), no trade can take place, despite differing private values. The buyer recognizes that she cannot profit from any deal the seller would offer. Similarly, the seller would make a loss at any price that the buyer would accept. This result is usually referred to as the "no trade theorem" and was first demonstrated by Milgrom and Stokey (1982).

[10] Of course, one must be careful that this choice of $p(x, y)$ satisfies the monotonicity conditions in (8) and (11). A sufficient condition to assure this is that $x + F(x)/f(x)$ and $y - (1 - G(y))/g(y)$ are both increasing functions.

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$$H = (1 + \lambda)(1 - \alpha)(y - x) - \lambda(1 - y + x).$$

Thus, the optimal mechanism is of the form: $p(x, y) = 1$ if and only if $y \geq x + \Delta$, where $\Delta > 0$. The IR constraint in (14) can be written:

$$\int_0^{1-\Delta} \int_{x+\Delta}^1 [(1 - \alpha)(y - x) - (1 - y) - x]dydx = 0.$$

Evaluation of this double integral leads to the equation:

$$(1/3)(2 - \alpha)(1 - \Delta)^2(\Delta + 0.5) - 0.5(1 - \Delta)^2 = 0,$$

which implies:

$$\Delta = 1.5/(2 - \alpha) - 0.5 = 0.5(1 + \alpha)/(2 - \alpha).$$

In short, the optimal mechanism calls for trade if and only if $y \geq x + 0.5(1 + \alpha)/(2 - \alpha)$. In turn, the ratio of the realized trading gains ($\pi_s + \pi_b$) to the maximal gains (π^*) is $(9/2)(1 - \alpha)^2/(2 - \alpha)^3$. This ratio approaches zero as α goes to one. Finally, we note in passing that the optimal mechanism calls for trade in exactly the same circumstances that trade occurs in the linear equilibrium of the SD procedure. Thus, in the special case of symmetric, uniformly distributed signals, the SD procedure's linear equilibrium is ex ante efficient.

4. Concluding Remarks

The seller-offer procedure, e.g., a posted price, is the most widely used method for making transactions in modern economies. Indeed, when the item for sale involves purely *personal* values for the agents ($\alpha = \beta = 0$), the seller prefers to name a take-it-or-leave it price over any other pricing mechanism.[11] By naming the price, the seller can exercise monopoly power even though this risks the loss of beneficial agreements (and produces accompanying inefficiencies).

The present paper extends the analysis to consider transactions having significant common-value elements. Using the direct revelation approach, Proposition 2 shows that increasing the degree of common values inevitably leads to reduced bargaining efficiency.[12]

This result would predict that a sale of a piece of art, for example, is less likely when it is valued for the sake of investment (i.e., when a common-value element is important) than when it is valued for the sake of personal enjoyment.

The use of the seller-offer procedure becomes more problematic as common-value elements increase. The split-the-difference procedure suggests that it offers significant efficiency advantages relative to the seller-offer mechanism. A principal advantage is that it capitalizes on both sides' private information, and hence delivers beneficial sales more frequently. Enjoying equal footing with the seller, the buyer unambiguously prefers the SD method to a seller take-it-or-leave-it offer. Given a sufficient degree of common values, even the seller benefits on average from the SD method.

[11] For a formal proof, see Williams (1987). In addition, as Riley and Zeckhauser (1983) show, the seller avoids being drawn into haggling with subsequent buyers by committing to a posted price.

[12] Vincent (1989) and Evans (1989) demonstrate analogous results for sequential bargaining: that dependent values lead to delayed agreements (and hence inefficiency).

Appendix A

We characterize $P(x)$ and $R(x)$ for x and y uniformly and independently distributed on [0, 1]. We will use the more general notation $v_s = Ax + By$ and $v_b = Cx + Dy$ with $A, B, C, D > 0$. With these coefficients and uniform distributions Equation (5) becomes

$$r'[(A - C)x + (B - 2D)r + D] - Cr + C = 0. \tag{A1}$$

Suppose the seller makes a subrange offer. Without loss of generality the seller may assume $0 \leq p/D \leq 1$. The seller's expected profit is

$$\int_{p/D}^1 [p - (Ax + By)]dy = D\frac{p}{D}\left(1 - \frac{p}{D}\right) - \left[Axy + \frac{1}{2}By^2\right]_{p/D}^1$$

$$= D\frac{p}{D} - D\left(\frac{p}{D}\right)^2 - Ax - \frac{B}{2} + A\frac{p}{D}x + \frac{B}{2}\left(\frac{p}{D}\right)^2$$

$$= \frac{B - 2D}{2}\left(\frac{p}{D}\right)^2 + (Ax + D)\frac{p}{D} - Ax - \frac{B}{2}. \tag{A2}$$

We assume $0 \leq R(x) \leq 1$. If the seller offers p below his range we may assume

$$0 \leq p \leq P(0) = DR(0) \quad \text{or} \quad 0 \leq p/D \leq R(0).$$

The Subrange Condition. In a perfect equilibrium, at $x = 0$ the seller's original offer $P(0) = DR(0)$ must be optimal; therefore, the RHS of Equation (A2) over the domain $0 \leq p/D \leq R(0)$, must reach its maximum at $R(0)$. This is clear if $R(0) = 0$ so suppose $R(0) > 0$. Suppose, moreover, $B - 2D < 0$ since the other case will take care of itself. Then we need

$$\frac{D}{2D - B} \geq R(0), \quad \text{or equivalently,} \quad \frac{D^2}{2D - B} \geq P(0),$$

which holds if $R(0) = 0$. It follows that

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$$\frac{D + Ax}{2D - B} \geq R(0),$$

and for all x in [0, 1] the maximum is at $R(0)$, which puts p inside the range of P.

In sum, in equilibrium, we need R a function on [0, 1] satisfying Equation (A1) and $R'(x) \geq 0$ on [0, 1]. If $B - 2D < 0$ we also need $D/(2D - B) \geq R(0)$, and the Above-Range condition

$$R(1) \geq \frac{A + D - C}{2D - B} \quad \text{or} \quad R(1) = 1.$$

Note that if in fact $B - D < 0$ then our constraint on $R(0)$ precludes R equal to the constant function 1. Thus, there exist some prices at which the buyer will buy some of the time.

If $A = C$ the solution to Equation (A1) is

$$Cx = (B - 2D)r + (B - D)\ln|r - 1| + k.$$

Otherwise, to solve the differential equation in (A1), we use the change in variables:

$$s = x - \frac{D - B}{A - C},$$

and $t = r - 1$. Then Equation (A1) becomes

$$t'[(A - C)s + (B - 2D)t)] - Ct = 0.$$

Let $t = vs$. Then $dt = vds + sdv$ and

$$(vds + sdv)[(A - C)s + (B - 2D)vs] - Cvsds = 0,$$

$$(A - C)vds + (B - 2D)v^2ds - Cvds + [(A - C) + (B - 2D)v]sdv = 0,$$

$$[(A - 2C)v + (B - 2D)v^2]ds + [(A - C) + (B - 2D)v]sdv = 0,$$

$$\frac{ds}{s} + \frac{(A - C) + (B - 2D)v}{(A - 2C) + (B - 2D)v}\frac{1}{v}dv = 0. \qquad \text{(A3)}$$

If $A - 2C = 0$, then the solution is

$$s = \left[\frac{B - 2D}{C}\ln|t| + k\right]t.$$

In the generic case $A \neq 2C$, Equation (A3) becomes

$$\frac{ds}{s} + \frac{A - C}{A - 2C}\frac{dv}{v} - \frac{B - 2D}{A - 2C}\frac{Cdv}{(A - 2C) + (B - 2D)v} = 0,$$

$$\ln|s| + \frac{A - C}{A - 2C}\ln|v| - \frac{C}{A - 2C}\ln|(A - 2C) + (B - 2D)v| + k_1 = 0,$$

$$\frac{A - C}{A - 2C}\ln|vs| - \frac{C}{A - 2C}\ln|(A - 2C)s + (B - 2D)vs| + k_1 = 0,$$

$$\frac{A - C}{C}\ln|t| - \ln|(A - 2C)s + (B - 2D)t| + k_2 = 0,$$

$$(A - 2C)s + (B - 2D)t - k_3|t|^{(A-C)/C} = 0, \quad \text{or}$$

$$s = k|t|^{(A-C)/C} - \frac{B - 2D}{A - 2C}t. \qquad \text{(A4)}$$

This is the counterpart of Equation (6) in the text.

Now we worry about which values of k in these solutions yield Nash equilibria, and which yield perfect equilibria. Suppose $D > B$ and $A \neq C, 2C$, so that Equation (A4) applies. The useful part of these curves resembles a parabola open in the increasing x direction in the x-r plane. As we vary k we get a nested family of curves. For k equal to a specific value—call this specific value K to be defined later—the curve is tangent to the r-axis. For $k > K$ the curve lies to the right of the r-axis, so we do not get a function defined for x in the positive neighborhood of 0. (See Figure A1.) Since R is increasing, we need the part of the curve above where the tangent is vertical. For $k < K$, this part of the curve intersects the r-axis too high for our subrange conditions to be met. For $k = K$ the subrange condition is met exactly. Similar arguments apply when $A = C$ or $A = 2C$.

We define K as follows: Set d/dt of Equation (A4) equal to 0, set $t = (B - D)/(2D - B)$ and let K be the solution for k.

If

$$\frac{D - B}{A - C} \leq 1 \quad \text{and} \quad A > C,$$

define R by Equation (A4) for

$$\frac{B - D}{2D - B} \leq t < 0,$$

and $R(x) = 1$ for

$$\frac{D - B}{A - C} \leq x \leq 1.$$

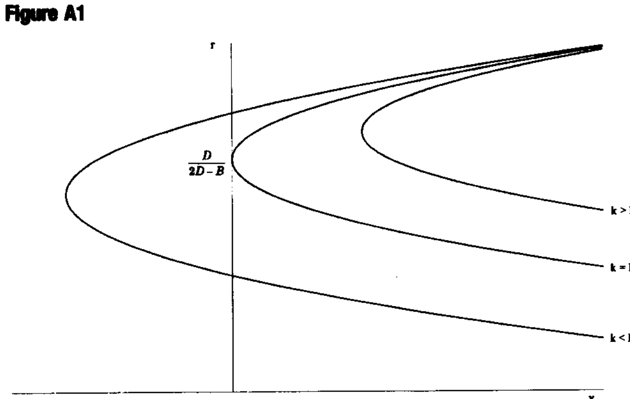
The pieces fit, but not smoothly, for $A > C$. This is acceptable since for fixed x, using equation (A1), we see that $A(x - z)R'(z)$, the derivative with respect to z of the seller's expected profit when he offers price $P(z)$, is 0 on the lower interval and has the proper sign on the upper interval and is continuous on the entire interval for z.

Appendix B
The Split-the-Difference Procedure

Derivation of Differentiable Strategies.

Let $s = S(x)$ denote the seller's equilibrium offer function, and let $h(s)$ denote the density function of these offers over the interval $[\underline{s}, \bar{s}]$. Let $b = B(x)$ and $j(b)$ over $[\underline{b}, \bar{b}]$ denote similar entities for the buyer.

Figure A1



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Finally, let $S^{-1}(\ )$ and $B^{-1}(\ )$ denote the respective inverse functions. The seller's conditional expected profit is:

$$\pi_s(s, x) = \int_s^{\bar{b}} [(b + s)/2 - [x + \alpha B^{-1}(b)]j(b)db.$$

Thus, the seller's first-order condition is:

$$\partial\pi_s(s, x)/\partial s = -sj(s) + (1 - J(s))/2 + xj(s) + \alpha B^{-1}(s)j(s) = 0.$$

Let $w \equiv B^{-1}(s)$. It follows that $s = B(w)$ and $J(s) = G(B^{-1}(s)) = G(w)$. In turn,

$$j(s) \equiv dJ(s)/ds = g(B^{-1}(s))/B' = g(w)/B'(w).$$

After making these substitutions, the seller's first-order condition becomes:

$$0.5B'(w)(1 - G(w)) + [S^{-1}(B(w)) + \alpha w - B(w)]g(w) = 0. \quad \text{(B1)}$$

In turn, the buyer's conditional expected profit is:

$$\pi_b(b, y) = \int_{\underline{s}}^{b} [y + \beta S^{-1}(s) - (b + s)/2]h(s)ds.$$

Therefore, the buyer's first-order condition is:

$$\partial\pi_b(b, y)/\partial b = yh(b) + \beta S^{-1}(b)h(b) - H(b)/2 - bh(b) = 0.$$

Let $z \equiv S^{-1}(b)$. It follows that $b = S(z)$ and $H(b) = F(S^{-1}(b)) = F(z)$. In turn,

$$h(b) \equiv dH(b)/db = f(S^{-1}(b))/S' = f(z)/S'(z).$$

After making these substitutions, the buyer's first-order condition becomes:

$$0.5S'(z)F(z) - [B^{-1}(S(z)) + \beta z - S(z)]f(z) = 0. \quad \text{(B2)}$$

Depending on the distributions $F(\ )$ and $G(\ )$, these linked differential equations may or may not have closed-form analytical solutions. For instance, linear solutions exist if F and G are of the form: $k(x - \underline{x})^{\gamma}$, where the parameters are arbitrary. For arbitrary distributions, the equations can be solved by numerical methods.

Appendix C
The Direct Revelation Mechanism

The Individual Rationality Constraint.

To derive the IR constraint in (14), we proceed as follows. From (3) we can write the seller's conditional expected payoff as

$$\pi_s(x) = \pi_s(1) - \int_x^1 P(s)ds.$$

In turn, the seller's ex ante expected payoff is

$$\pi_s \equiv E_x[\pi_s(x)] = \pi_s(1) - \int_0^1 \int_x^1 P(s)dsdF(x).$$

After integrating the second integral by parts and using the definition of $P(x)$, this can be rewritten as

$$\pi_s = \pi_s(1) + \int_{\underline{y}}^{\bar{y}} \int_0^1 F(x)p(x, y)g(y)dxdy. \quad \text{(C1)}$$

In turn, the buyer's ex ante expected profit can be written as

$$\pi_b \equiv E_y[\pi_b(y)] = \pi_b(\underline{y}) + \int_{\underline{y}}^{\bar{y}} \int_{\underline{y}}^{y} Q(s)dsdG(y)$$

$$= \pi_b(\underline{y}) + \int_{\underline{y}}^{\bar{y}} \int_0^1 [1 - G(y)]p(x, y)f(x)dxdy. \quad \text{(C2)}$$

The sum of the players' ex ante profits is simply:

$$\pi_s + \pi_b = \int_{\underline{y}}^{\bar{y}} \int_0^1 [(1 - \alpha)y - (1 - \beta)x]p(x, y)f(x)g(y)dxdy. \quad \text{(C3)}$$

To see this, note that for a particular realization of estimates, the sum of the players' profits is:

$$(p - v_s) + (v_b - p) = v_b - v_s = (1 - \alpha)y - (1 - \beta)x.$$

Using (C1), (C2), and (C3), we can rewrite the IR condition in (13') as:

$$\int_{\underline{y}}^{\bar{y}} \int_0^1 \{[(1 - \alpha)y - (1 - G(y))/g(y)]$$

$$- [(1 - \beta)x + F(x)/f(x)]\}p(x, y)f(x)g(y)dxdy \geq 0.$$

This is the expression in (14).

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