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Optimality in a World of Progress and Learning

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Optimality in a World of Progress and Learning

THE PROBLEM

We consider the case of an immortal craftsman who learns as he continues to turn out more of a product via a given technique. His labour is the only factor of production; it is supplied at the rate of one unit per time period. His objective is to maximize total output over a long time span. He is fortunate to be living in a world where there is continual improvement in the techniques available to him. His problem is to decide when it is desirable to switch from an old to a new technique. A change will result in the loss of the productivity gained through prior learning, but will give him the opportunity to employ a more advanced technique. As he gains experience, his productivity with the new technique will surpass what it would have been had he continued with the old.¹

FORMULATION AND ANALYSIS

Output is a function of the input of the one factor, labour. The rate of flow of labour input is fixed; therefore, it is possible to express the output produced via a single technique as a function of the length of the interval during which the technique is employed. Technological progress is treated as an increase in output per unit of input. Letting y represent output, and x both labour input and the time interval during which a particular technique is in use, we have,

$$y = kf(x), \quad \dots(1)$$

where k is a multiplicative factor representing the level of technology for the technique.

Progress occurs continuously at the rate r . The k for a given technique will depend upon the period in which it is introduced. Representing k for a technique introduced at time 0 as k_0 , the k for a technique introduced at time t will be $k_0 e^{rt}$. The relationship between technological progress and learning is such that for any technique the ratio of output in the t_i th to that in the t_j th period is independent of k . If this were not the case, the length of the optimal interval would vary over time.

We are considering a simple case of a situation most common in economic dynamics. Whenever there exists an achievable growth rate as great as or greater than the rate of discount (in this instance zero), the optimal plan will be one which achieves the highest possible growth rate. In the case at hand, any plan which switches techniques at regular intervals will result in output (averaged over the interval during which a single technique is in use), growing at the maximal rate r . If many plans can achieve the highest growth rate, the object is to select the one among those plans which can sustain the highest rate of output along the growth path. The gains to be derived from raising the level of output along the growth path will in time exceed any fixed amount. Therefore, losses sustained in getting to the growth path are not of consequence, and straightforward methods of maximization will enable us to identify the optimal plan.

Our task is to find the optimal length of period, n , during which the craftsman should stick to a single technique. If the length of the total period under optimization, m , is great (for our immortal craftsman it is infinite), we need not be concerned with problems of divisibility by n , and n can be considered to be independent of m . As we proceed with the optimization, m will drop out of our calculations.

¹ The athletically interested reader might prefer to think in terms of a sportsman who changes his techniques (e.g., style or equipment) as he improves his game. Every change entails a temporary setback which is more than made up for with time.

Total output over an m unit span, with a change in technique every n periods, is equal to

$$kf(n) + e^{rn}kf(n) + e^{2rn}kf(n) + \dots + e^{r(m-n)}kf(n). \quad \dots(2)$$

output
output
output
in 1st n
in 2nd n
in last n
periods
periods
periods

Summing the series, we have

$$kf(n) \left[\frac{e^{r(m-n)} - \frac{1}{e^{rn}}}{1 - \frac{1}{e^{rn}}} \right]; \quad \dots(3)$$

this simplifies to

$$kf(n) \left[\frac{e^{rm} - 1}{e^{rn} - 1} \right]. \quad \dots(4)$$

To maximize, we differentiate with respect to n and set the derivative equal to 0.¹ We get

$$\frac{k(e^{rm} - 1)}{(e^{rn} - 1)^2} [f'(n)(e^{rn} - 1) - f(n)re^{rn}] = 0. \quad \dots(5)$$

To satisfy this equation, we set the expression within the heavy brackets equal to 0. Therefore,

$$\frac{f(n)}{f'(n)} = \frac{1}{r} \left[1 - \frac{1}{e^{rn}} \right]. \quad \dots(6)$$

It will not be possible, in general, to derive a closed-form solution for this differential equation. It will often be convenient to proceed, as might a craftsman, by using graphs to find the optimal value for n .

AN EXAMPLE

In his pathbreaking article, "The Economic Implications of Learning by Doing", Professor Arrow refers to empirical data concerning the production of airframes:

... the number of labor-hours expended in the production of an airframe (airplane body without engines) is a decreasing function of the total number of airframes of the same type previously produced. Indeed, the relation is remarkably precise; to produce the N th airframe of a given type, counting from the inception of production, the amount of labor required is proportional to $N^{-1/3}$. This relation has become basic in the production and cost planning of the United States Air Force ... [1; p. 156]

In the terms of our model, the production of each type of airframe involves the use of a single technique. To switch the type of airframe produced is to lose the benefits of past learning, but to put oneself in the position to reap the fruits of technological progress.

Our model abstracts from reality in that it employs a one-factor production function.² We write cost solely as a function of labour input. The relationship we consider is of the form

$$x = cy^a. \quad \dots(7)$$

¹ We are assuming $f(n)$ is differentiable throughout. Without a knowledge of the specific form of $f(n)$, it is not possible to say whether second order conditions are fulfilled. In all the examples discussed in this essay, second order conditions are satisfied.

² Given certain not unreasonable assumptions, it would not be difficult to extend our results to a two-factor model. We believe, however, that the single-factor model captures the most interesting aspects of the situation we consider.

where x is cost in labour units and y is output. Learning is represented by a decreasing marginal cost of output which implies that a is less than 1. We invert this function so that in the manner of equation (1) we can write output as a function of input;

$$y = kx^b. \quad \dots(8)$$

It can be seen that b equals $1/a$ and k equals $(1/c)^{1/a}$. Now substituting our specific $f(x)$ into equation (6), we get

$$\frac{n}{b} = \frac{1}{r} \left[1 - \frac{1}{e^{rn}} \right]. \quad \dots(9)$$

The airframe example gives a value of $2/3$ for a which corresponds to a value of $3/2$ for b . For the purposes of this example, we choose an arbitrary r equal to $\cdot 05$. For this value of r , the optimal n is $17\cdot 5$, a switch in techniques should be made every $17\cdot 5$ time units.¹

Output in the unit time period after a switch will be less than 38% of what it would have been had the old technique been continued. This is a familiar phenomenon in dynamic situations. In the interest of efficiency, it is often desirable to make a short-term sacrifice in absolute output so as to get to the growth path sooner or, as in this case, more often.

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- [1] Arrow, K. J. "The Economic Implications of Learning by Doing", *Review of Economic Studies*, **29** (1962), 3.

¹ Applying a discount rate to output would increase the length of the optimal period. With a discount rate equal to the growth rate, the objective is to maximize average discounted output per unit of time. In an example given in a fleshier version of this paper, it was found that the move from a zero to a 5% discount rate increased the optimal technique interval by about one-eighth.