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Source: *Journal of Risk and Uncertainty*, Vol. 5, No. 3 (1992), pp. 205-216
Published by: Springer
Stable URL: https://www.jstor.org/stable/41760662
Accessed: 23-05-2020 22:17 UTC

Journal of Risk and Uncertainty, 5:205-216 (1992)

Multidimensional Bargains and the Desirability of Ex Post Inefficiency

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Key words: bargaining, asymmetric information, second best, incentive compatible, revelation principle

Abstract

In multidimensional bargaining situations where individuals possess relevant private information, say about preferences, allocational efficiency is a central concern. Even if there is no squabbling over distribution—for example, if contingent commitments on allocations can be made before private information is secured—honest revelation comes only by sacrificing efficiency. Indeed, the incentive-compatible, second-best outcomes generally require that some allocations be off the contract curve (ex post inefficient). The potential for recontracting, by ruling out such inefficient allocations and the second-best equilibria they support, would hurt matters further.

Uncertainty plays a central role in bargaining. The uncertainty may be shared, in the sense that neither player knows how future events will unfold, but both players have common information and agree on probabilities. Information asymmetries may also enter—for example, if B is uncertain about A's preferences, or if A does not know B's information about the state of the world. Our concern here is with such asymmetric situations.

Resources are often destroyed irrationally or inadvertently in the hurly-burly of bargaining under asymmetric information. Players attempting to secure a more favorable bargain may delay, carry out threats, or misrepresent in a manner that kills a deal.

Our concern, however, is with inevitable rather than irrational or transaction-cost bases for inefficiency. If only a single object is to be sold, even if the bargainers are fully rational masters of game theory and there are no frictional costs to bargaining, when information is asymmetric all mechanisms must sacrifice efficiency; that is, some desirable trades do not get made.[1]

This article extends the discussion of bargaining under asymmetric informational to multidimensional problems, those that involve more than one good. Such problems include, for example, virtually all bargains over jobs or commercial contracts. Though

*Support was received from the Decision, Risk and Management Science Program of the National Science Foundation. Rachel Algaze Croson provided able assistance; referees provided helpful comments.

there may be deep equivalences between multidimensional and single-good bargaining problems, there are strong differences in qualitative flavor. When there are several goods, the players have a strong interest in achieving allocative efficiency, placing the right goods in the right hands. In principle, as Raiffa (1982, p. 131) phrases it, the players "can cooperate in order to enlarge the pie they will eventually have to divide." But can they? Such a cooperative concern still competes with the players' efforts to increase their personal shares. For example, if A knows that B values x_1 a great deal, A may as a bargaining ploy pretend to attach a high value to x_1 himself. This stratagem, however, creates the danger that A may ultimately end up with x_1 instead of x_2, if goods are divisible, with the lion's share of the wrong one.

We derive results for situations where the players provide unverifiable information about the state of the world. To induce honesty, various incentive-compatibility constraints must be met. To meet such constraints most economically, assuming they are binding, ex post efficiency must be sacrificed. Our analysis shows in detail why ex ante efficiency forces the prescribed division of goods off the contract curve for some player reports, regardless of the procedure through which agreement is reached.

1. The problem

This analysis addresses multidimensional bargaining with divisible goods and private information. First-best outcomes—those that could be reached if all information were costlessly shared—are beyond reach. We are concerned with both ex ante efficiency (reaching the incentive-constrained frontier in expected utility space) and ex post efficiency (ending up on the contract curve for the actual state of the world).

Our results apply no matter how the distributive aspects of the bargaining problem are resolved. The prescribed division may follow some game-theoretic solution such as those of Nash or Shapley. It may be externally imposed. Or norms may be so well established that both players know "where it will come out," and take that result as a guideline. In an operational sense, we assume that the outcome of the bargaining process is a point on the ex ante efficient frontier for expected utility.[2]

The players have private information that they report to guide the allocation. Such information may relate to private preferences (how I feel about a better job title as opposed to a better salary), or to some common-value issue (e.g., in designing an oil deal, my assessment of the prospects of finding oil). We assume that the allocation depends solely on what the players say, rather than any statistical or direct verification. We invoke the Bayesian common priors assumption, posit that both players adhere to von Neumann–Morgenstern utility, and seek ex ante efficient outcomes.[3]

Consider a simple case in which A is the only observer and only reporter. A and B are dividing up some infinitely divisible foodstuffs and athletic supplies. Let s be the state of the world, which for this illustration relates only to A's preferences. If $s = 1$, A has an intense passion for both types of goods, with a special tilt toward the athletic supplies. If $s = 2$, he prefers the foodstuffs, but his utilometer registers only weakly. B likes a balanced mix of the two types of goods; his preferences do not depend on s. The first-best division would give A virtually everything if $s = 1$, and very little if $s = 2$. Knowing this, and knowing that the allocation would be implemented, A would simply state that $s = 1$, no matter what the reality might be.[4]

There is no costless way out of the incentive-compatibility problem. If we wish to capitalize on the information that a player will reveal, we must make it worthwhile for him to reveal it. In the example at hand, we must decrease the utility of A's take given the report $s = 1$, and increase it for the report $s = 2$, until when the true state is 2, A would just be willing to say $s = 2$ rather than $s = 1$. Obviously, first-best efficiency is sacrificed.[5] There is no reason, however, why the ex post division of the goods should not be efficient. Or is there?

2. The loss of ex post efficiency

Our central result is negative. When the first-best solution is not incentive compatible, an ex ante efficient, second-best solution must be sought. Unhappily, some or all of the allocations under the second-best solution will be ex post inefficient. In other contexts, ranging from signaling (Spence, 1973) to targeting transfers (Nichols and Zeckhauser, 1982), economists have found that an efficiency price must be paid to induce different types of individuals to sort themselves. Thus, high-quality workers are forced to buy an education whose cost exceeds the value that it adds to human capital; truly needy citizens are put through ordeals to qualify for welfare payments. In each instance, the reason is that it would be more costly for the other type (respectively, the low-quality worker and the fraudulent welfare recipient) to engage in the same activity. In the bargaining arena, the division of payoffs given certain truthful reports must be deformed away from efficiency so as to make false reports less attractive in alternative states of the world.

Suppose the following structure and functions are common knowledge (that is, the state incorporates all relevant private information). The state has a discrete probability distribution. The probability of state s is $f(s)$.

The supplies of the goods are normalized so that for each state there is one unit of each good.[6] In state s, if A receives allocation x (a vector), B receives the complement. Denote A's utility by $u(x, s)$, and B's utility by $v(x, s)$, both as functions of A's allocation. Let u_i and v_i be partial derivatives with respect to the ith component of x. Assume $u_i \geq 0 \geq v_i$ everywhere for all i.

An allocation is first-best iff it maximizes $\sum_s f(s)u(x, s) + \lambda \sum_s f(s)v(x, s)$ for some λ. If s could be costlessly observed, straightforward maximization would provide an answer. The allocations here, however, depend on unverifiable information provided by A, which imposes an incentive-compatibility constraint on his payoffs.

An allocation x is ex post efficient (on the contract curve for state s) iff it maximizes $u(x, s) + \lambda v(x, s)$ for some λ (which may depend on s); that is,

$$u_i(x, s)/v_i(x, s) \text{ is the same for all } i \text{, say } -\lambda(s). \tag{1}$$

It is first-best iff in addition $\lambda(s)$ is the same for all s.

The contract curve for state s is the locus in (u, v) space given by condition (1) as λ varies. Unless the contract curves for different values of s coincide at the point of interest, the first-best outcome requires that the allocation depend on s.

Suppose A knows s and will receive an allocation $X(r)$ depending solely on his report r. The allocation rule X is incentive compatible iff $u(X(r), s) \leq u(X(s), s)$ for all r, s. Our question is how to proceed if a desired (first-best efficient) allocation rule is not incentive compatible. We shall see that when the incentive-compatibility constraint is binding, ex ante efficiency under this constraint will ordinarily require some ex post inefficiency.[7] Intuitively, if A prefers the state-1, allocation in state 2, then the allocation when A reports 1 must consider A's utility if 2, as well as A's and B's utilities if 1, and must negatively weight A's utility if 2 to deter him from falsely reporting 1 in state 2. In general, this moves us off the contract curve for state 1, since that curve takes into account only utilities in state 1. Thus, in this second-best situation, the "optimal" allocation if A announces 1 will be ex post inefficient. Throughout we assume that the ex ante efficient frontier is concave, implying that efficiency does not require randomization.

2.1. *Two possible states*

When only two states are possible, the ex post efficient allocations lie on just two contract curves, and the first-best efficient allocations are pairs of points, one on each of these same curves. There are four basic cases:

1. The desired allocation rule X is incentive compatible iff A prefers $X(1)$ to $X(2)$ in state 1 and $X(2)$ to $X(1)$ in state 2, where "prefers" allows indifference and we assume honesty under indifference. Then A's indifference contours and the contract curves create a bow-tie configuration as in figure 1, perhaps with multiple crossings. (The dashed curve is an indifference contour for A for state 1; the solid curve is an indifference contour for A for state 2. The curves and contours lie in a space that has as many dimensions as there are commodities.)
2. A may prefer $X(1)$ in both states, strictly in state 2. The indifference contours form a band, as in figure 2.
3. A may prefer $X(2)$ in both states, strictly in state 1. This is a mirror image of figure 2.
4. A may prefer $X(1)$ in state 2 and $X(2)$ in state 1. This inverts the bow-tie in figure 1.

Case 2 leads to concern with ex ante efficiency subject to the constraint $u(x, 2) = u(y, 2)$, where x and y are A's allocations when he reports 1 and 2, respectively. Letting $q = f(2)/f(1)$ (i.e., the ratio of the states' probabilities), we have the Lagrangian

$$u(x, 1) + qu(y, 2) + \lambda[v(x, 1) + qv(y, 2)] + \mu[u(y, 2) - u(x, 2)], \tag{2}$$

where λ is the relative weight on B's utility and μ is the shadow price of incentive compatibility when $u(y, 2) \geq u(x, 2)$ is binding. Hence the first-order conditions are, for all i:

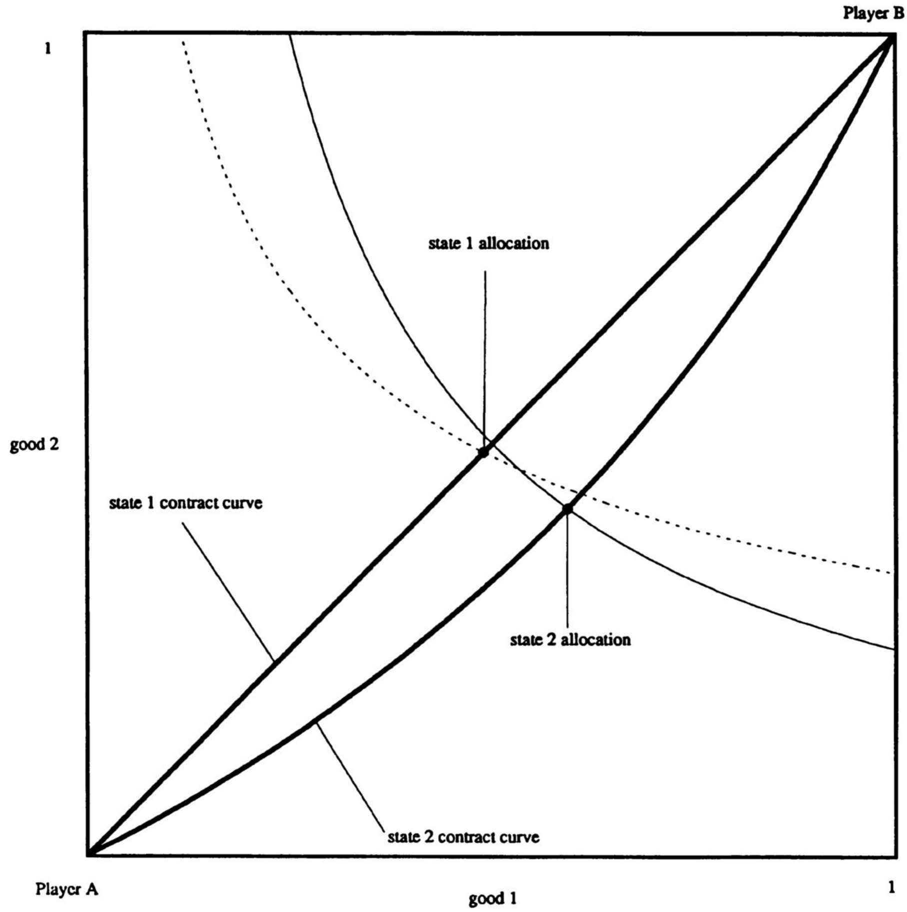

Figure 1. Desired allocation is incentive compatible.

$$u_i(x, 1) + \lambda v_i(x, 1) - \mu u_i(x, 2) = 0, \tag{3}$$

$$u_i(y, 2) + \lambda' v_i(y, 2) = 0 \text{ where } \lambda' = \lambda q/(q + \mu). \tag{4}$$

Since $\mu = 0$ would give the unconstrained optimum, we must have $\mu > 0$. Thus condition (4) makes y ex post efficient in state 2, but B's utility is underweighted ($\lambda' < \lambda$). By condition (3), x will be ex post efficient in state 1—$u_i(x, 1)/v_i(x, 1)$ the same for all i—iff $u_i(x, 1)/u_i(x, 2)$ is also the same for all i, in which special case B's utility is overweighted in state 1. This leads to the following proposition.

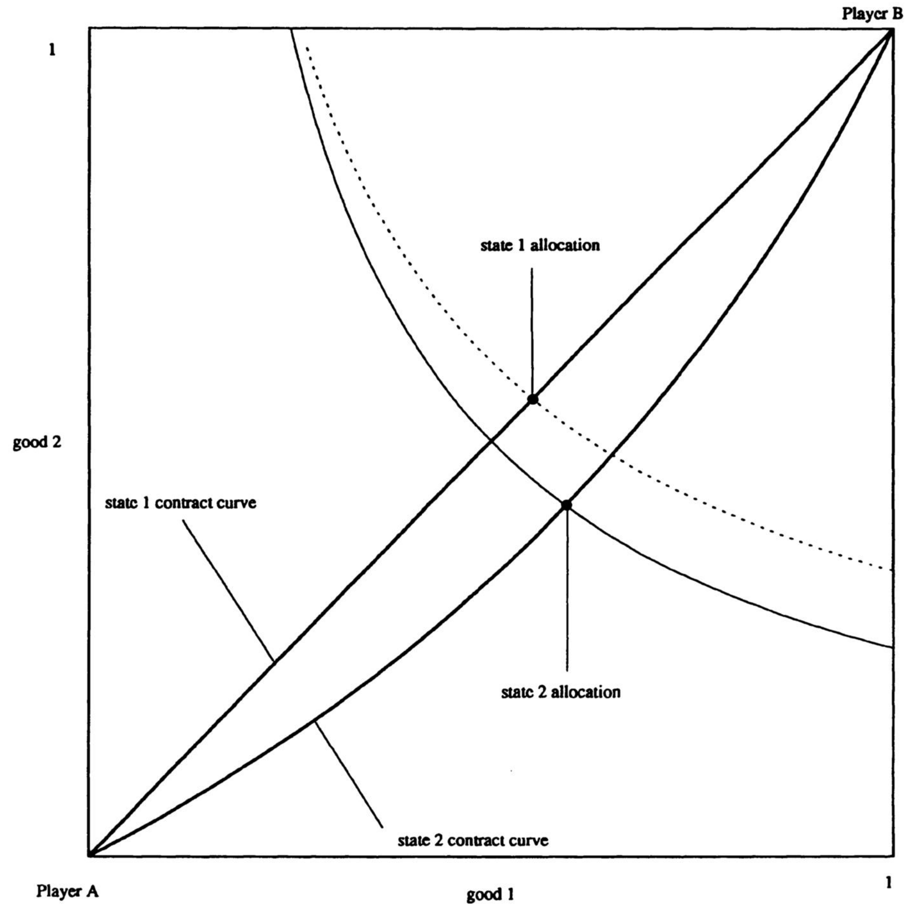

Figure 2. A prefers state 1 allocation in both states.

> *Proposition 1.* In case 2, an ex ante (incentive-compatible) efficient allocation (x, y) replaces the unconstrained efficient allocation $X(2)$ by another point y on the state 2 contract curve more favorable to A. In general, it replaces $X(1)$ by a point x off the state 1 contract curve—that is, by an ex post inefficient point.

In the very special situation where $u_i(x, 1)/u_i(x, 2)$ is the same for all i, $X(1)$ is replaced by a point on the $s = 1$ contract curve less favorable to A. Ordinarily, however, this condition does not hold, and x is not ex post efficient in state 1.

Case 3 reverses states 1 and 2 in case 2.

In case 4 the compatibility constraint is binding in both states, giving the Lagrangian

$$u(x, 1) + qu(y, 2) + \lambda[v(x, 1) + qv(y, 2)] + \mu[u(y, 2) - u(x, 2)] + \nu[u(x, 1) - u(y, 1)] \tag{5}$$

where ν is the shadow price of incentive compatibility when $u(x, 1) \geq u(y, 1)$ is binding. The first-order conditions become

$$(1 + \nu)u_i(x, 1) + \lambda v_i(x, 1) - \mu u_i(x, 2) = 0, \tag{6}$$

$$(q + \mu)u_i(y, 2) + \lambda q v_i(y, 2) - \nu u_i(y, 1) = 0. \tag{7}$$

Now x will be post efficient in state 1 iff either $\mu = 0$ or $u_i(x, 1)/u_i(x, 2)$ is the same for all i. Similarly, y will be ex post efficient in state 2 iff either $\nu = 0$ or $u_i(y, 2)/u_i(y, 1)$ is the same for all i. Even in these situations, it is not obvious whose utility is downweighted. The important point, however, is that these conditions are very special, and ordinarily neither x nor y will be ex post efficient.

2.2. *Many states*

Again, if the desired first-best allocation is efficient and incentive compatible, it will of course be ex ante and ex post efficient. Otherwise, however, ex ante efficiency entails some number of binding incentive-compatibility constraints of the form

$$u(x_s, s) = u(x_t, s), \tag{8}$$

where x_s and x_t are the allocations to player A if he reports s and t, respectively. The Lagrangian is

$$\sum_s f(s)[u(x_s, s) + \lambda v(x_s, s)] + \sum_{s,t} \mu_{s,t}[u(x_s, s) - u(x_t, s)] \tag{9}$$

where $\mu_{s,t} = 0$ for $s = t$ and for all pairs (s, t) for which incentive compatibility is not binding (at the solution). The first-order conditions are that, for each i and s,

$$[f(s) + \sum_t \mu_{s,t}]u_i(x_s, s) + \lambda f(s) v_i(x_s, s) - \sum_t \mu_{t,s} u_i(x_s, t) = 0. \tag{10}$$

If all binding constraints (8) have the same value of t, say $t = 1$, then the situation is like case 2 above. Specifically, for $s \neq 1$, expression (10) becomes

$$[f(s) + \mu_{s,1}]u_i(x_s, s) + \lambda f(s) v_i(x_s, s) = 0; \tag{11}$$

x_s is ex post efficient in state s, but B's utility is underweighted by an amount depending on s. Note that the requirement for this case is not that all incentive incompatibilities have the form $X(1)$ is preferred to $X(s)$ in state s, but that when all incentive incompatibilities of this form are efficiently eliminated, no other false reports remain or become desirable.

2.3. *Both agents partially informed*

Suppose $s = (s_1, s_2)$ where A knows s_1 and reports t_1 while B knows s_2 and reports t_2. Constraints now apply to both agents. Unless one of the agents has no binding incentive-compatibility problem and the other's incentive incompatibility is of the type leading to equation (11), the situation in all states will be like equation (3), not equation (4). By implication, ex post efficiency can hold only if the marginal utilities satisfy a very special condition.[8]

3. Example

Consider an example in which the two possible states are equally likely. A's utility function in state 1 is

$$u(x, 1) = x_1^{.4} x_2^{.4},$$

and in state 2 is

$$u(x, 2) = x_1^{.2} x_2^{.1}.$$

B's utility function is state independent,

$$v(z) = z_1^{.4} z_2^{.4},$$

now expressed in terms of B's allocation $z = (z_1, z_2)$.

There is one unit of good 1, and one of good 2 in each state. Let x_{ij} be the amount of good i going to A in state j; B receives the complement. The contract curves are found by equating the marginal rates of substitution for the two players. They are $x_{11} = x_{21}$ for state 1, and $x_{22} = x_{12}/(2 - x_{12})$ for state 2, as shown in figure 3. To find an ex ante efficient outcome, we employ a positive λ, and find the x_{ij}'s, for $i,j = 1,2$, to maximize

$$.5[u(x_{11}, x_{21}, 1) + u(x_{12}, x_{22}, 2)] + .5\lambda[v(1 - x_{11}, 1 - x_{21}) + v(1 - x_{12}, 1 - x_{22})].$$

For $\lambda = .8$, the maximization yields points C and D as the divisions for states 1 and 2, respectively. The unbroken curves are two of A's indifference curves for state 2; the short dashes give one of A's curves for state 1.

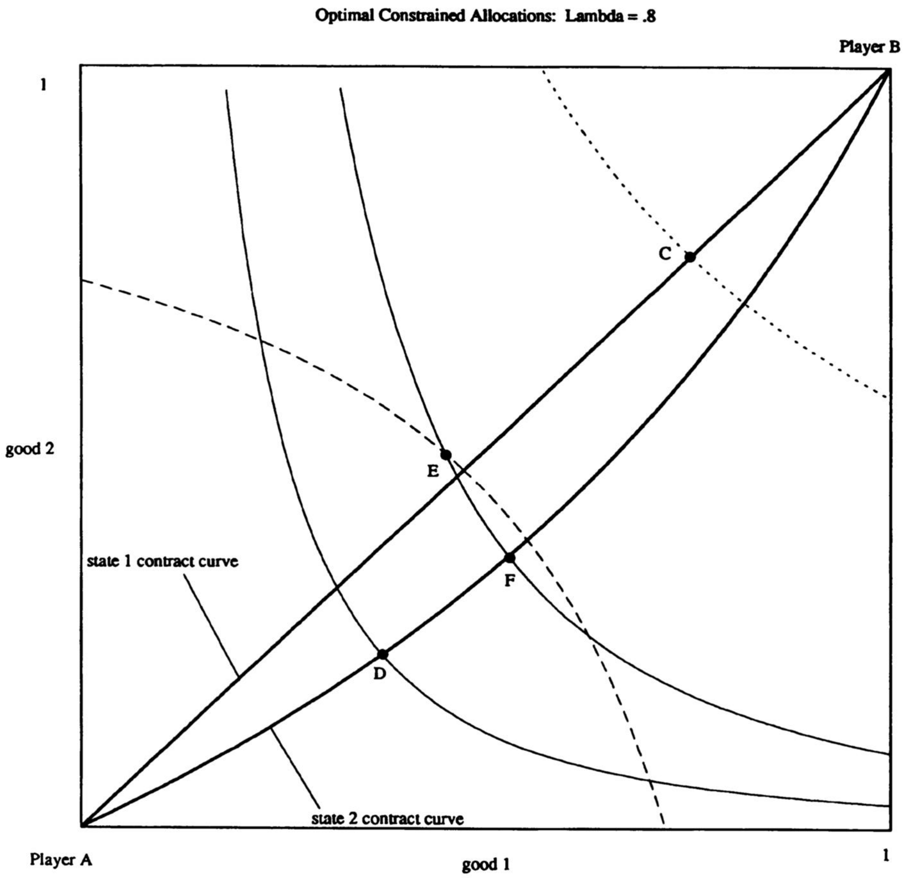

Figure 3. Ex ante efficient allocation is off state 1 contract curve.

The challenge, as shown in the diagram, is that A, who knows whether state 1 or 2 applies, would always report 1, since he prefers that allocation even if state 2 pertains. To induce him to be honest, we define new points following equations (2), (3), and (4). The state-2 second-best point slides up along that contract curve to F. The state-1 second-best point, E, maximizes a three-component weighted average, with A and B's utilities if $s = 1$ weighted positively, and A's utility if $s = 2$ weighted negatively. Given the third component, E lies off the state-1 contract curve, as shown in the figure. The remaining condition is that A is indifferent between E and F in state 2. There is a lens-shaped area between B's indifference curve through E (shown with long dashes) and A's state-1 indifference curve through that point (not shown) that contains all points that are ex post superior to E in state 1.

4. Qualifications

4.1. Randomization

We did not allow randomized allocations, which would expand the strategy space and might be valuable in many circumstances. For example, assume that the incentive-compatibility problem with the first-best allocation is that A will always announce state 1. Assume further that A is much more risk averse in state 2 than in state 1. Then, we can deter A from falsely announcing 1 when 2 applies by randomizing the allocation employed, given an announcement of 1. (We assume B is not strongly risk averse in state 1, so the utility loss to him from randomization does not overwhelm its truth-inducing advantages.)

If either player is risk averse, assuming the other is not risk seeking, the use of randomized allocations by itself is ex post (to the announcement) inefficient. Except in knife-edge cases—such as those involving risk neutrality and straight Pareto possibility frontiers—we would expect the optimal second-best strategy to randomize among allocations that are themselves ex post inefficient. Despite these theoretical niceties, nonrandomized allocations seem more realistic for real-world bargaining situations.

4.2. Recontracting

Say that an optimal set of allocations has been selected among those constrained to be incentive compatible. Assuming the constraint was binding; barring exceptional cases, for some states of the world the allocation will be ex post inefficient. Proceeding forward, information was acquired and revealed; the specified allocation was inefficient. Would it not make sense to recontract now?

If it is known that there will be recontracting, however, the incentive for honest revelation will be lost under the foregoing allocation. The whole problem collapses in upon itself. The fundamental difficulty is not that recontracting would be undesirable if the state was reported truthfully (or misrepresented). Indeed, in some instances recontracting from an inefficient allocation would be desirable whichever true state applied. Rather, the difficulty is that with recontracting, expected payoffs can no longer be adjusted so flexibly to achieve incentive compatibility. Recontracting buys ex post efficiency only by sacrificing second-best outcomes.[9] Our results show that to achieve ex ante efficiency requires binding commitments against recontracting.

5. Conclusions

Allocational efficiency, we know from real-world experience, is frequently not fully secured in multidimensional bargaining. Do not professors, for example, sometimes end up with smaller teaching loads than they would willingly accept, while being paid less than they would like? We investigated whether such outcomes result because players have an incentive to distort relative preferences as a means to increase their share of the pie. We found, unfortunately, that even when there was no squabbling over distribution, inefficient ex post allocations are likely to result. Indeed, strategic moves off the contract curve are required to assure honest reporting at the minimum cost in efficiency. When the first-best outcome is not incentive compatible, alas, inefficient ex post outcomes become a general ingredient of second-best solutions to multidimensional bargains.

Notes

1. Chatterjee and Samuelson (1983) first showed this for reservation prices drawn from a uniform distribution. Myerson and Satterthwaite (1983) showed this for more general distributions, building on the revelation principle and an argument that traders will have an incentive to lie about their types; some beneficial bargains do not get struck. Our analysis shows that in a continuous multidimensional setting, bargains will not be made in the most efficient way.

   Chatterjee, Pratt, and Zeckhauser (1978) show that efficiency can be achieved if player participation is assured, either by commitment before types become known, or because players would wish to participate for any values of their private information. Their method applies, for example, to the common situation where the seller quotes a price, and the other player accepts or rejects. Paying the seller the expected benefit that the potential buyer receives from his quote induces the seller to quote his reservation price. Efficiency is therefore achieved. This expected externality principle also works when the buyer quotes a price and the seller chooses whether to accept, or when both players name a price and the difference is split. It also applies to a range of more general problems (Pratt and Zeckhauser, 1987).
2. If the fight over any division of spoils were included, matters would be worse, perhaps much worse. The difficulty we deal with is that the players may not accurately report their information: doing so may not be incentive compatible (D'Aspremont and Gerard-Varet, 1979).
3. The solution could insist on ex post efficient allocations, but that would further restrict the second-best solution, producing a third-best solution. In addition, a bargaining solution could be imposed separately for each state of the world, say if there were a concern for ex post fairness. This would move us into the fourth-best world of woolly thinking.
4. In a repeated-play game, with A drawing his type each trial, we could restrict A to saying $s = 1$ only a certain percentage of the time.
5. If there were some external currency whose value did not change with the state of the world, then we could charge A for making alternative statements that influence the allocation. If money failed to work, perhaps an hour of dishwashing would, for example. Budgets could even be balanced, but for some draws of private information, players might choose not to participate. See footnote 1.
6. Since we produce a negative result, we need not consider more complex situations where supplies are random variables and allocations may depend on supply levels.
7. It is an easy case of the revelation principle (see Holmstrom, 1977; Rosenthal, 1978) that any implementable allocation rule is equivalent to one that is incentive compatible.
8. If $s = (s_0, s_1, s_2)$ where neither agent knows s_0, then the problem can be reduced to the case just discussed by expecting out s_0 with respect to its conditional distribution given (s_1, s_2).
9. See also footnote 6. The recontracting issue has an intriguing backward causality feature, which is reminiscent of Newcomb's Paradox. There are two boxes, A and B. A contains \$1000; B has either \$1 million or nothing, you do not know which. You have two possible actions: 1) take what is in both boxes, or 2) take only what is in box B. What is in B depends on the prediction a Superior Being made about what you would do. If he predicted 1), there will be nothing in B. If he predicted 2), there will be \$1 million. In either case, you are rewarded for making the choice the Being predicted. But the prediction has already been made; the money is already in the boxes. Would it not be prudent to make sure you get the \$1000, just in case?

In our situation, the reporter(s) have already lied or told the truth. We can no longer deter them. Does it not make sense to trade in the direction of superior outcomes, no matter what the reality proves to be?

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