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THE EFFECT OF THE TIMING OF CONSUMPTION DECISIONS AND THE RESOLUTION OF LOTTERIES ON THE CHOICE OF LOTTERIES[1]

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WHEN AN INDIVIDUAL chooses an occupation, a job, or a portfolio for his investments, he is in effect selecting among lotteries on wealth. His wealth will define his range of choice on consumption allocations, the ultimate source of his utility. In addition the individual must make consumption decisions, some of which will have long-run implications. The purpose of this note is to clarify the importance of the timing of decisions on consumption allocation and the resolution of the selected wealth lotteries.

Consider a consumer with a consumption bundle represented by the vector x, who has a von Neumann-Morgenstern utility function, $u(x)$, over such bundles. The individual faces the price vector p. Before the resolution of his chosen lottery, his wealth, w, is a random variable.

Imagine, first, that the consumer selects his preferred consumption bundle after w is known. He will therefore select x to vield

$$\max_{\{x:px\leqslant w\}} [u(x)] \tag{1}$$

Let the maximum value of the above problem be denoted by $v(w)$, which emphasizes the dependence of the maximum on w.

Thus $v(w)$ is a derived von Neumann-Morgenstern utility function for money. The consumer may use it to evaluate alternative lotteries on wealth. He should select that lottery that offers him the highest expected value for $v(w)$.

This note considers the case when some of the consumer's consumption decisions have to be made *before* the outcome of the wealth lottery is known. We shall show that in general under these conditions, no cardinal utility function on wealth exists that correctly evaluates lotteries for the consumer. Moreover, the consumer will have a more conservative response to uncertainty than he would were lotteries resolved before any consumption decisions were taken. The sense of the last statement can be made clear with the aid of a simple example.

1. INCREASED AVERSION TO RISKS

Consider an individual with the two-commodity utility function $x_1^{.25}x_2^{.25}$, who is confronted with the price vector $p = (1, 1)$. If lotteries are resolved before any consumption allocations are undertaken, his utility function on wealth will be $v(w) = w^{.5}$. A lottery that offers wealths of 1000 or 2000 each with probability one half will have a certainty equivalent of 1457.

Change the rules of the game and require the individual to make a decision on the amount of x_1 to be purchased before the lottery is conducted. The lottery is now much less attractive, in the sense that the individual would now accept a much smaller certain amount in lieu of it. The lottery's certainty equivalent when there must be a preresolution consumption allocation is 1376.

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This illustrates a general phenomenon. Requiring an individual to make consumption allocations (or decisions of any sort) before a lottery is resolved will make him substantially more eager to face a lottery which (in an approximate sense) offers little variance in its potential payoffs. (To an external observer who is unaware of any reversal in the timing of lottery outcomes and consumption decisions, the individual will appear to be substantially more risk averse.) The reason of course is that the presence of significant uncertainty as to the magnitude of his payoff makes it impossible for him to make pre-lottery decisions that will turn out to be efficient ex post.

2. NON-EXISTENCE OF A UTILITY FUNCTION

A consumer required to make some pre-lottery consumption allocations will encounter a further difficulty. He will find it impossible, in general, to derive a von Neumann-Morgenstern utility function on wealth. The substitutability axiom of the von Neumann-Morgenstern theory will not apply. Interchanging equivalent lotteries in a compound lottery will affect its expected utility. Markowitz [2], Drèze and Modigliani [1], and Mossin [3] have discussed this difficulty elsewhere.

Consider an example. We stated above that the two lotteries $\ell_1 = (.5, 1000; .5, 2000)$ and $\ell_2 = (1.0, 1376)$ offer equal expected utility when the individual must make his x_1 purchase decision before the lottery outcome is known. Examine the compound lottery, $\ell_c = (.5, \ell_1; .5, \ell_2)$, where the individual is required to select x_1 before any part of ℓ_c is resolved. The compound lottery will be inferior to either of its two components. If he were facing ℓ_1, the individual would select $x_1 = 658$. If he were facing ℓ_2, he would select $x_1 = 688$. When he faces the compound lottery, he will pick an x_1 which will be less attractive for at least one of the component lotteries than the x_1 values just mentioned. This is the source of the loss in performance.

3. THE NON-EXISTENCE PROBLEM IN A MORE GENERAL FORM

The consumer faces a four part decision problem. These are in chronological order: (i) First he selects a wealth lottery, ℓ_j, from among a number of alternatives, $j = 1, \ldots, n$. (ii) He restricts his consumption decision by selecting one of a collection of sets $X_1, \ldots, X_m$, from which he will later choose his final consumption bundle. (iii) Nature selects his wealth, w, based on his choice of lottery. (iv) The consumer selects his consumption bundle from X_i to maximize utility, subject to the usual budget constraint $px \leqslant w$.

The standard analysis is to work backwards. At step (iv) the consumer knows ℓ_j, X_1, and w. He must select $x \in X_i$ to maximize $u(x)$ subject to the requirement that $px \leqslant w$. Denote this maximum by

$$\max_{\{x \in X_i : px \leqslant w\}} [u(x)]. \tag{2}$$

Before step (iii) the choice of w is uncertain. Hence the consumer must view w in (2) as a random variable and take the expectation of (2) where the probability distribution of w depends on the choice ℓ_i at step (i). Denote this expectation by

$$E_{w|\ell_j} \{ \max_{\{x \in X_i : px \leqslant w\}} [u(x)] \}. \tag{3}$$

The value (3) depends on ℓ_j and X_i. At step (ii) the consumer must choose X_i to maximize (3) Denote this maximum by

$$\max_i E_{w|\ell_j} \{ \max_{\{x \in X_i : px \leqslant w\}} [u(x)] \} \tag{4}$$

The value of (4) depends only on the choice of ℓ_j. At step (i) he must choose ℓ_j to maximize (4). This maximization yields

(5) $$\max_j \max_i E_{w|\ell_j}\{\max_{\{x \in X_i : px \leqslant w\}} [u(x)]\}.$$

If on the other hand steps (ii) and (iii) were interchanged and the choice of X_i could be made conditional on knowledge of w, instead of (5) we would have

(6) $$\max_j E_{w|\ell_j}\{\max_i \max_{\{x \in X_i : px \leqslant w\}} [u(x)]\},$$

and the expression in heavy braces could be interpreted as the induced utility $v(w)$ of wealth w. It is obvious that in any concrete example the value (5) is less than or equal to the value in (6).[2] Of course, if the choice of i in (6) does not depend on the value of w, then the values of (5) and (6) will be identical.

It should be noted that if ℓ_1 and ℓ_2 are significantly different but have the same expected utility, the consumer will prefer either ℓ_1 or ℓ_2, to all (but degenerate) compound lotteries involving ℓ_1 and ℓ_2. The knowledge of which compound lottery he will face enables the consumer to avoid additional uncertainty. In the specific context of our earlier example, this argument demonstrates that when a non-trivial commitment to a consumer choice is forced prior to the resolution of a lottery, the certainty equivalent of that lottery declines.

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REFERENCES

[1] DREZE, JACQUES, AND F. MODIGLIANI: "Epargne et consommation en avenir aléatoire," Cahiers du Seminaire d'Econometrie, 1966.

[2] MARKOWITZ, H. M.: *Portfolio Selection: Efficient Diversification of Investments*. New York: John Wiley and Sons, 1959.

[3] MOSSIN, JAN: "A Note on Uncertainty and Preferences in a Temporal Context" (Communication), *American Economic Review*, 59 (1969), 172–173.

[2] To return to the simple two-good case, it can be shown that there is a cardinal utility index if and only if the utility function has the form $u(x, y) = f(y + g(x))$, or $u(x, w - x) = f(w - x + g(x))$. This form clearly implies that the optimal choice of the first good, x, is independent of wealth.