DETERMINING THE QUALITIES OF A PUBLIC GOOD—A PARADIGM ON TOWN PARK LOCATION

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Where should we build the town park? This query represents the general question a community encounters whenever it considers the provision of a public good: what qualities should the good possess? In the theoretical literature for the most part, public goods decisions are usually thought of as those of choice of scale: how much of it should we provide? The central argument of this paper is that qualitative differences matter as well. For example, town parks at different locations will enter individuals' utility functions in different ways.[1]

Choice of location is obviously meant to represent choice along any qualitative dimension or dimensions. Should the music at the annual concert lean toward rock or traditional? In allocating our school budget, what is the appropriate mix of real estate expenditures and instructional expenditures? And within the instructional budget, how much stress should we place on science as opposed to the humanities and social sciences?

In a significant array of public choice situations, the process that sets budgetary amounts is divorced from the allocation decision within each budget. Though it is not essential to the argument, the analysis below assumes such a division of budgetary authority. This simplifies exposition; it also seems to reflect a pattern commonly observed in the real world. A legislature will frequently set expenditure limits, with the executive or administrative branch determining exactly the ways in which the funds will be spent. Voters may approve a school bond issue, with the school board deciding how best to employ the funds for education. Indeed, we frequently settle on projects and the means through which they will be financed, and leave the determination of qualities for future consideration, perhaps by a different decision-making body.

I. THE MODEL

To keep the analysis simple, let us consider a most primitive model designed to answer our opening question about the park: where should

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1. One interpretation would be that the town parks at different locations were different public goods. This perfectly reasonable construction would imply that the choice of the qualitative characteristics of a public good was in fact the selection of one among a group of competing public goods.

it be built? Following the tradition of resurgent welfare economics, group welfare is treated as a positively associated function of individual welfares. Thus to maximize group welfare it will be necessary to reach a constrained maximum with respect to the individuals' utilities, to reach a Pareto optimum.

There are but two participants in our model, Adams and Brown, though the model could equally well deal with many. These two make up the total population of the town and quite naturally live at opposite ends of Main Street. The public good on which they are to make their decision, the town park, is fully described by a single characteristic, its location along Main Street. There is no room for choice on other dimensions such as size, facilities, etc.

The location, indicated by m, is given on a linear scale with Adams' house indicated at 0 and Brown's at 1.[2] The cost of the park and the tax shares to provide it are givens; they are independent of location. The after-tax resources of Adams and Brown are r_A and r_B. Their respective utility functions are $U_A(m, r_A)$ and $U_B(m, r_B)$.

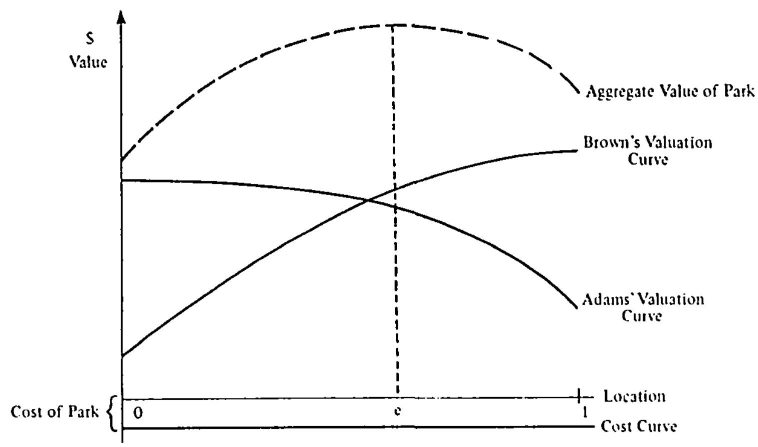
Each individual indicates how much he would pay for the park in any location; close to home is best. For the present, assume that these valuations are negligibly affected by resource levels over the ranges of resources considered. If m were a private good, this would be equivalent to assuming that income effects were inconsequential. This assumption enables us to employ individuals' valuation curves to summarize all the information on their preferences that is required for optimization. The optimization procedure is straightforward for the situation shown in Figure 1. The dashed curve shows the total net value of the park at each location. It is derived by adding the three curves vertically and reaches its maximum at e, the efficient location for the park.[3]

In this simple case, where efficiency is the only concern, the park has a clearly defined optimal location. This model provides the point of departure for three classes of complexities now to be considered: distribution of welfare, possible preference for lottery outcomes, and preferences for particular types of societal outcomes.

2. The qualitative characteristics of some public goods are not ordinarily indicated on a cardinal scale (such as miles from a particular location). A nation's foreign policy, surely a public good by traditional criteria, cannot be measured on any convenient spectrum or spectra. Even if a summary scale could be developed, say left-right orientation of policy, individuals would have to define cardinal rankings over this scale. Imposing this requirement is equivalent to asking housewives how much they would pay for higher grades of produce, or asking professors to define the strength of their preference for A students over those receiving B's and C's.

3. What is being employed here is a variant of the Kaldor-Hicks criterion; distributional considerations are set aside till later pages. See Little [7]. With convex (though possibly backward bending or reverse image) indifference curves, marginal analysis will lead to the optimum. In a later Section, we shall see that convex-preferences in this context may guarantee much less than we might expect when income effects do play a role.

Figure 1



II. DISTRIBUTION OF WELFARE—SELECTING AN ETHICALLY ATTRACTIVE POINT ON THE PARETO FRONTIER

In matters of public choice, distribution of welfare is a consideration quite apart from efficiency. In the private competitive market, initial endowments define the point that is reached on the Pareto frontier, i.e., they determine the distribution of welfare. There is no direct equivalent to initial endowments where the provision of public goods is concerned. Numerous outcomes may be efficient; of these, many may represent a Pareto improvement over the status quo. Additional criteria are required if we are to select among the available points on the Pareto frontier. Two possible criteria are (1) that the point be ethically attractive, and (2) that it be universally accepted.

If we could impose anonymity on these individuals, the two criteria could be combined. Prior agreement would be reached on a point on the Pareto frontier, before anyone knew his identity. The objective would be to determine an outcome that was not only desirable, but equitable in the sense that each individual's welfare was accorded the same weight.[4]

If Adams and Brown did not know their own identities, but knew that there was a fifty-fifty chance that they would end up as either man, they might be able to reach an agreement on where to locate the park. Their

4. Rawls [12] employs the notion of an original position where all participants stand behind a "veil of ignorance," not knowing who was to be who in future society. Rawls analyses the ethical principles individuals in such a position would choose to guide their society. I have proposed elsewhere [16] that such individuals, if rational, would seek to maximize their expected future utilities, assigning probability $1/n$ to the likelihood they would end up as any one of the n future members of society. Rawls does not endorse the expected utility approach. Nevertheless, our discussions together suggest that in many expectable situations, following his principles will lead to the maximization of expected utility.

problem would be particularly straightforward if they had identical utility functions with respect to valued variables. They would pick a location and a set of lump sum transfers[5] that would maximize their expected utility, given a one-half chance of being either.[6] In the town park example, they would reach a symmetrical rule. Either the town park would be in the center of town, or there would be two points that were indifferent from the standpoint of maximizing expected utility. In the two-point case, they might employ a lottery to choose between the two.[7]

Problems arise if the situation is not perfectly symmetrical. Initial resource endowments may be unequal, and lump sum transfers may be impossible to make or be limited in some way. But these complications just mean that the optimal solution may not be a symmetrical one.

A significant difficulty arises if the two individuals have different utility functions. With identical functions there is no need to worry about linear transformations; for example, if the goods endowment [*park one mile away, $937*] is given a utility of 80 by Adams' function, it would be given the same by that of Brown. With different utility functions, however, there can be no assurance on this point.

What we really want to do is find some weights that should be assigned to the two utility functions. If linear transformations were carried out, the weights would be changed accordingly. (Adding a constant to either utility function would require no changes.) Quite often the unit that is invoked is the utility that is derived from a marginal dollar. The selected outcome is the one that maximizes total dollar benefit. This approach, if it is understood to have any normative significance, implies that a dollar to A yields the same utility as a dollar to B. Economists have learned to scorn this type of interpersonal comparison. But the most important drawback of the total dollar approach to welfare is that it does not have equity

5. See McGuire and Aaron [9] for a model that is easily extended to cope with situations where redistributional capabilities are severely limited. Their analysis demonstrates that the way a public good is paid for may affect the optimal amount to provide. Structural limitations on cost-sharing schemes for public goods—for example, tax codes that pre-specify sharing ratios—are not examined in this paper.

Barlow [9] and Peltzman [10] provide instructional illustrations of the efficiency losses engendered when the form of provision for public services is restricted. Barlow demonstrates that local school taxation is structured so that expansion of expenditures helps the rich and hurts the poor. He concludes that alternative forms of local taxation, perhaps income based, could produce a situation where marginal benefits more closely reflect marginal burdens. Benefits-based taxation of this sort expands the Pareto frontier. Peltzman, focusing on the higher education scene, describes the inefficient outcome that has resulted because redistributional measures were limited to transfers in kind.

6. See Raiffa [11] for a discussion of cardinal utility functions defined over more than one attribute.

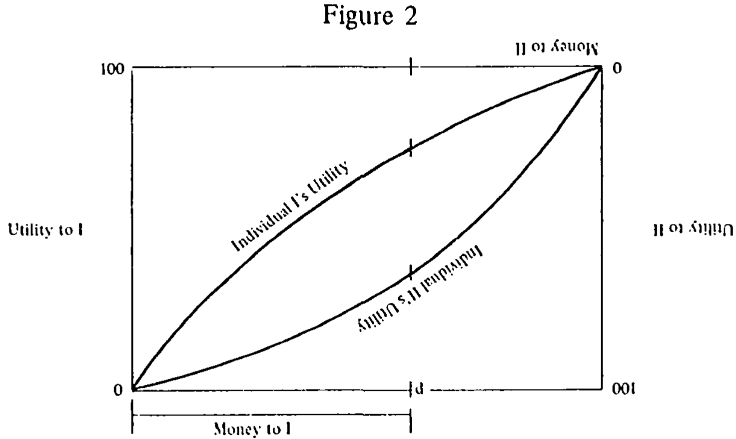
7. This actually would not be necessary. The fifty-fifty chance of being either man insures that starting with identity unknown will prevent inequity or inefficiency.

as its motivating force.

Establishing a Common Utility Unit Through Lotteries. One way to motivate the standard unit problem on an equity basis is to define the common unit in terms of a lottery. It may be impossible to make interpersonal comparisons between Adams and Brown, but we may agree that, in the absence of knowledge as to who will be who, the opportunity to raise oneself from some defined bad outcome to some defined good one should be treated the same way for each of the two. The argument can be made on the ground that if one does not know any better, there is no reason to assume a bias one way or another. An equity argument could also be used to endorse this approach.

In a one-dimensional scheme this plan might work as follows. Transform the two individuals' utility function so that a utility of 100 is assigned to \$10,000 and a utility of 0 to \$5000. Now divide the money to maximize the sum of the two utility functions, in effect so as to maximize the expected utility of an individual who has a fifty-fifty chance of being either man. This method will not in general lead to a symmetric outcome. The procedure for finding the optimum is shown in Figure 2. At point d, the slopes of the two utility functions are equal, their sum is maximized.

Figure 2



The problem grows more complicated if there is more than one attribute. The good point and the bad point have to be defined on each attribute. Let us demonstrate this procedure for the park example. The good point will be *[park 0 units away (just-at-house), \$10,000]*. The bad point will be *[park 1 unit away, \$5000]*. The two individuals' utility functions are scaled so these points take on the values 100 and 0. In-between points are defined by scaling them between these two outcomes. A point assigned a value of 25 would be one such that the individual would be indifferent between staying at that point for certain and taking a lottery which gave him a 25 percent chance of winding up at the good point but a 75 percent chance of losing and ending up at the bad point.

An Example with Utility Functions that Display Risk Aversion. Consider a numerical example with what might be called normal utility functions. Both Adams and Brown are risk-averse with respect to gambles on either argument of their utility function, the value of the other argument being fixed. Their indifference curves are convex. These two facts together guarantee that their utility surfaces will be concave; chords connecting two points on the surface will lie below the surface. The hypothetical utility functions are

$$U_A = r_A{}^4(1.1 - m)^4 \quad \text{and} \quad U_B = r_B{}^4(.2 + m)^4$$

They are relevant for the values $r_A \geqslant 0$, $r_B \geqslant 0$, and $0 \leqslant m \leqslant 1$.

The utilities for the good *[0 away, $10,000]* and bad *[1 away, $5000]* points would be:

	Good Point	Bad Point
Adams, U_A	41.358	12.011
Brown, U_B	42.823	15.849

To normalize these good and bad point utilities to 100 and 0, Adams' utility function is transformed by multiplying it by 3.408 and subtracting 40.929. Brown's utility function is transformed by multiplying it by 3.709 and subtracting 58.757.

In this numerical example, both utility functions are separable in a way that guarantees that the optimal location of the park and the proportion in which after-tax resources are divided do not depend upon the sum of initial resource endowments.[8] With other types of utility functions, this independence will not exist. For the utility functions given here, the optimal values of r_A, r_B, and m are:

8. It may not be possible to make all the lump sum transfers that one might wish. The costs shares may to some extent be predetermined as a function of the public good provided. For example, with $r_A + r_B = \$15,000$, the alternate divisions of funds specified the following optimal locations for the park:

$r_A = \$4,500$	$r_B = \$10,500$	$m = .670$
$r_A = \$7,500$	$r_B = \$7,500$	$m = .495$
$r_A = \$10,500$	$r_B = \$4,500$	$m = .314$

The utility functions are such that the more money either man has, the more he gets his way. Other utility functions would give different results.

	$r_A + r_B = \$10{,}000$	$r_A + r_B = \$15{,}000$	$r_A + r_B = \$25{,}000$
r_A	3961	5943	9904
r_B	6039	9057	15096
m	.585	.585	.585

The Optimization Process. Let U_A^* and U_B^* represent the utility functions that have been normalized for the good and bad points, and R be the total resources of Adams and Brown after paying for the park. The optimum is the

$$\underset{m, r_A, r_B}{\text{MAX}} \quad U_A^* + U_B^*$$

subject to

$$r_A + r_B = R$$

The constraint is internalized by writing $r_B = R - r_A$. The problem is thus to find the

$$\underset{m, r_A}{\text{MAX}} \quad U_A^*(m, r_A) + U_B^*(m, R - r_A)$$

Differentiating with respect to the controllable variables and setting the results equal to 0 yields

$$U_A{}^*_1 + U_B{}^*_1 = 0 \quad \text{and} \quad U_A{}^*_2 - U_B{}^*_2 = 0$$

The first condition is that the sum of the marginal utilities for moving the park must be zero. The second condition is that the two individuals have the same marginal utility for resources.[9]

Sensitivity to the Choice of Good and Bad Points. This scheme has an unfortunate arbitrary factor: the outcome will depend in general on the good and bad points selected. Consider, for example, the one-attribute situation where money was the only argument of the utility functions. Raising the value of the attribute at the good point in effect compresses the graph. Consider a two-person world in which one individual has a linear utility function, but the other is risk-averse. Raising the attribute

9. If the cost of the park varies with m, call it $C(m)$, the first condition becomes more complicated. It is

$$U_A{}^*_1 + U_B{}^*_1 - C'(m) \cdot U_B{}^*_2 = 0$$

The last term represents the loss (gain) of increased (decreased) cost from moving towards Brown multiplied by the marginal utility of money.

value at the good point will decrease the slope of the utility function by the fraction

$$\left[1 - \frac{\text{old utility of old good point}}{\text{old utility of new good point}} \right]$$

For the risk-neutral individual, this will be a greater decrease than for the risk averter. Raising the value of the argument(s) of the good point will thus work in favor of the risk averter and conversely.

In the park example there are two arguments to the utility functions, and the rescaling of good points changes the outcome in a slightly complex fashion. To see the magnitude of the effect, define a new good point to be *[park 1/2 unit away, $8000]*. (The old good point was *[park 0 units away $10,000]*.) The optimal values for $R = \$15,000$ are:

	Given Old Good Point	Given New Good Point
r_A	5974	6808
r_B	9026	8192
m	.583	.510

Any attempt to follow the suggested scheme, to assign two points for scaling utility functions and then to maximize expected utility, will suffer from the difficulty that the points assigned may affect the outcome. The question that must be asked is, is this worse than having no guide at all to help select the outcome? It would seem reasonable to hope that, with some experience with these methods, we might find acceptable ways to define the needed points. No economist has yet proved that we can never find an acceptable way to choose outcomes *as if* we could make interpersonal comparisons of utility.

Unequal Initial Endowments–An Arbitration Approach. If we are allocating resources to maximize a social welfare function, or to maximize a sum of various individuals' utilities, we need not be concerned with the way in which total initial resources were divided. (We assume utility is defined in a static context and does not depend on changes from past conditions.) In the real world, however, initial conditions often significantly affect outcomes.[10] They may also influence individuals' feelings about the equity of different outcomes.

Assume that Adams started with $10,000 and Brown with $25,000. Then normalizing their utility functions and maximizing the sum might well lead to a situation in which Brown was worse off than he was at the start. Such a reduction in welfare, however justified, might be unaccep-

10. Almost all arbitration and fair division schemes rely on initial conditions to determine desirable or equitable outcomes. See Luce and Raiffa [8].

table, particularly to Brown if he knew his previous position.

To deal with this problem and still maintain the spirit of our approach, we could employ scalings that have different good and bad points for the two individuals. With no loss in generality we can establish the starting points as the bad points. The outcome will then depend entirely on the good points selected. This will be a bargaining problem, although equity considerations may play a role in establishing the good points. The higher the other individual's good point, the better will be the outcome for Adams or Brown. One way of determining these good points, consistent with the previous approach, would be to set the good points as a fixed amount of improvement over the *status quo*, perhaps \$10,000 additional and the park 1/2 unit closer.

The Maximin Approach. The disparity in initial positions suggests another approach: the two individuals might agree to maximize the the minimum utility for either of them. With a maximin agreement, it will always be to an individual's advantage to have his good point as well as his bad set as high as possible. On both an equity and bargaining basis, we might like to see the bad points set at the *status quo*. This would guarantee that both would gain from any arrangement that represented an improvement over the *status quo*. Fixing the good points would once again represent a bargaining problem, although equity considerations might enter the picture. Establishing a common good point would work to the advantage of the initially disadvantaged individual. Further, the lower the common good point, the better off he would be.

The maximin criterion was introduced because it has some flavor of arbitration, something that is missed if expected utility is maximized. Yet, as we will see shortly, maximizing expected utility has other shortcomings; it may lead to a very unequal *ex post* outcome. A criterion that seems to give attractive outcomes in prospect may actually lead to results that in retrospect are seen as undesirable.

Even in situations in which starting positions are revealed, it may be highly desirable to bargain over criteria to determine the outcome rather than over the outcome itself. This moves the decision-making process to a more fundamental level, where personal interests are less easily identified. This is the basic conception of Buchanan and Tullock in *The Calculus of Consent*, where they argue that a constitution specifying social decision procedures must be formulated before social choice can proceed.

Addressing the debate to criteria rather than to outcomes has further virtues besides anonymity. Criteria can employ notions of equity; whereas points on the Pareto frontier are just that, points. The process of agreeing on criteria for choosing outcomes looks like a social-welfare-oriented procedure. To bargain over outcomes directly is to miss out on these supposedly virtuous underpinnings. Haggling on criteria rather than outcomes can have an additional benefit. If problems are complicated, both parties may be unable to perceive how the adoption of different criteria will affect them. In the absence of haggling, they might both agree to adopt criteria set I over criteria set II, even though the latter would have turned out to have been more favorable for one of them.

Directing the debate to criteria will have an additional advantage if the social choice criterion itself enters as an argument of individuals' utility functions. People may just prefer to live in a society that chooses outcomes one way rather than another. If so, the position of the maximin criterion would seem to be strengthened. Most individuals would likely be attracted to its orientation towards equality.

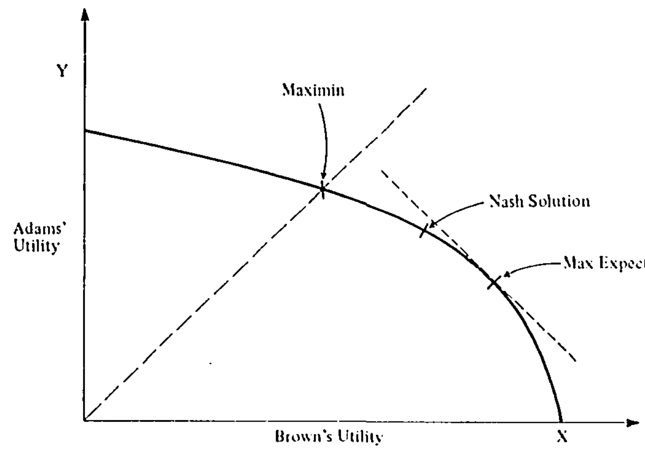
The expected-utility objection to this line of argument is that if criteria matter, they should be introduced into individuals' utility functions before maximization is undertaken. Meeting this objection may introduce a chicken-egg paradox. Use of the maximin criterion as the guide for social choice may yield higher expected utility than use of a criterion that is explicitly directed toward expected utility maximization.

Comparison of Outcomes Selected by Different Criteria. It is instructive to compare the outcomes selected by the maximize-expected-utility criterion and the maximin criterion with the Nash Solution, the best known of the arbitration outcomes.[11] Assume, as is usual, that the utility-possibility frontier is strictly concave and continuous. The outcome that maximizes expected utility for a randomly chosen individual is indicated by the point on the utility-possibility frontier where the slope is minus one. The maximin criterion will select the point where the utilities of the two individuals are equal. The Nash Solution, like the outcomes selected by the other criteria, requires an external reference; specifically, it requires a *status quo* point which is assigned a utility value of zero for each individual. In a bargaining context, it is natural to think of the mutual bad point employed by the other criteria as the *status quo* point for the Nash Solution. The Nash Solution will be indicated by the point on the utility-possibility frontier at which the ratio of the individuals' utility values (the one measured on the vertical axis over the one measured on the horizontal axis) is the negative of the slope.

It is easily seen that the Nash Solution will always lie between the maximin point and the point that maximizes expected utility; it will be less egalitarian than the former (farther from the 45° line), but more egalitarian than the latter. In the case in which any two of these points coincide, all three will coincide. At the maximin point the utility ratio is

11. See Luce and Raiffa [8].

Figure 3



1 and the absolute value of the slope is X. These values for the point that maximizes expected utility are, respectively, Y and 1. Either X and Y will both be greater than 1 or both be less than 1. As we move along the utility-possibility frontier from the maximin point to the point that maximizes expected utility, the utility ratio and the absolute value of the slope will move in opposite directions in continuous, strictly monotonic fashions. The point along the way at which these two values are equal is the Nash Solution.

The situations considered thus far have been fairly tractable. It has always been possible to identify the compromise outcome that would be selected by any particular social choice criterion. Two major difficulties arise: (1) the social choice criterion may select a non-compromise outcome, and (2) individuals may have preferences for societal outcomes that go beyond their concern for consumption bundles or the way in which societal outcomes are selected.

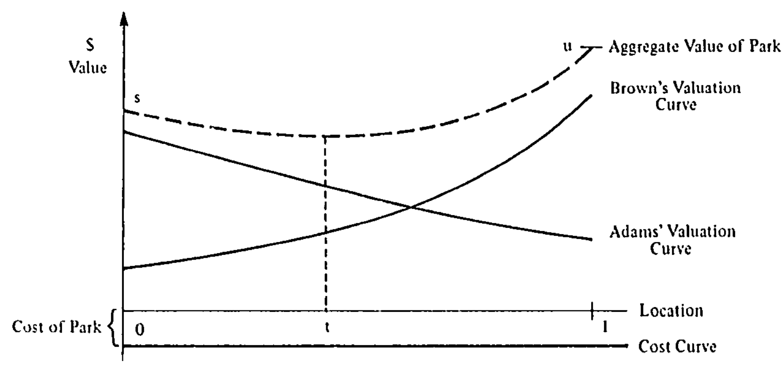
III. NON-COMPROMISE OUTCOMES

Non-convexity. Preferences may not be convex. In the town park example, convexity would require something like: the closer the location gets to home, the less one is willing to pay to move the park a unit closer.[12] This requirement does not seem to be particularly reasonable. Surely most individuals would pay more to bring a park one-half mile closer if it were a mile away rather than ten miles away. If so, the total.

12. This is slightly less than fully general, because monotonic rescalings of location are permitted. The more precise technical definition of convexity would require that for some such rescaling all individuals' marginal willingnesses to pay for movement decrease as the park moves closer to their desired locations.

valuation curves would bulge the "wrong way," producing a concave upwards Aggregate Value of the Park Curve.[13]

Figure 4



With the U-shaped aggregate value curve, traditional marginal optimization techniques may lead us astray. Whenever the initial point under consideration lies to the left of t, movement in the direction of rising valuation will eventually place us at the local optimum s, not the global optimum u. Still and all, a more sophisticated algorithm can be developed that will enable us to get to the efficient point. The convexity difficulties were stressed here, for they shall play a more significant role in the next portion of the analysis.

Difficulties When Utility Functions Display Risk Preference on a Variable. If individuals are risk preferrers with respect to gambles on an argument of their utility function, the outcome the criterion judges to be optimal may give very unequal utilities to the two individuals. It was argued above that when one of the "commodities" that is being provided is not a conventional good, but rather a qualitative aspect of a public good, it is quite reasonable to expect that individuals will not be risk averse with respect to gambles on favorable and unfavorable changes in the amounts of the commodities provided.

13. To test for convexity where there is no natural underlying cardinal scale, we might employ hypothetical gambles. If two individuals always prefer some compromise outcome to a lottery on extremes, there exists a scale for which their preferences are convex. Political mechanisms are not usually constructed to reveal whether individuals are risk preferrers with respect to movements along some spectrum. However, the institution of logrolling is to some extent a reflection that a decision that is bad for A but good for B, together with a decision that is good for A but bad for B, may be better for each of them than a pair which has a compromise on each decision. A "one for you, one for me" type of approach need not reflect exploitation of any third party. (It is often claimed that a situation was created in the French Third Republic in which two nearly incompatible societies survived in an intertwined fashion because they were willing to make political deals in this way.)

See Zeckhauser [15] for a discussion of majority rule procedures when individuals may be risk-preferrers for the set of available alternatives.

If the *status quo* is that the town park gets built at 1/2, both Adams and Brown might be willing to enter into a lottery which makes it 50-50 that it will be placed at either 0 or 1. This would seem to interfere with the type of in-between outcome that we generally expect to be optimal when individuals' preferences are convex.

The willingness to take fair actuarial gambles with respect to one argument of the utility function need not imply that one's indifference curves are not convex.[14] The utility functions

$$U_A = r_A^{.4}(1.1 - m)^{1.2} \text{ and } U_B = r_B^{.4}(0.2 + m)^{1.2} \qquad \text{for } 0 \leq m \leq 1$$

are utility functions for which Adams and Brown would be risk preferrers with respect to gambles on the location of the park, m, even though their indifference curves (relating after-tax resources and nearness to park as the valued variables) are always convex. As before, Adams would most like the park to be at 0, Brown would prefer a location at 1. However, with these particular utility functions they would both be risk averters with respect to gambles on money.

A Temporary Paradox. Before we continue with our numerical example, consider a result that might at first appear paradoxical. If the individuals are risk preferrers with respect to gambles on the location of the park, any Pareto-optimal plan will always result in the location of the park at either 0 or 1. If the park were to be placed at an in-between location n, $0 < n < 1$, Adams and Brown would agree to a lottery which had an n probability of placing the park at 1 and a $1 - n$ probability of placing the park at 0. In agreeing to abide by the results of the lottery, the individuals would be securing a Pareto improvement with respect to expected utility.

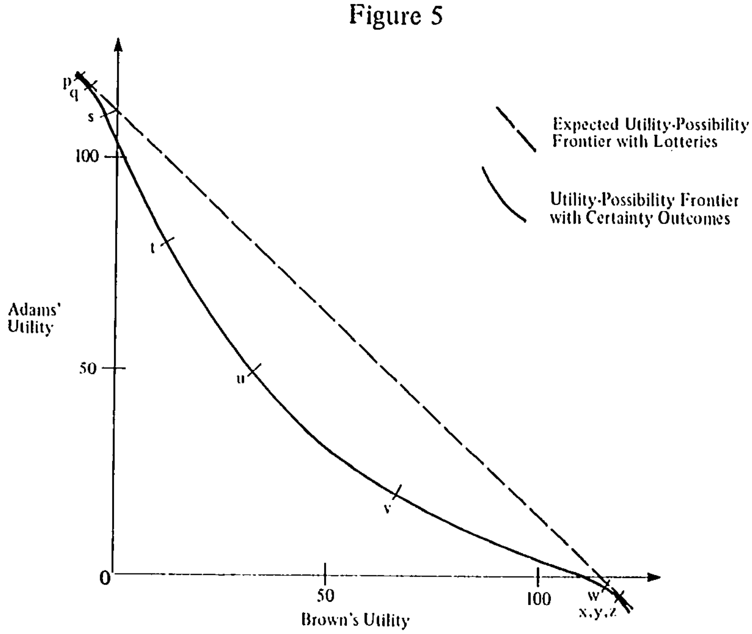
But there is an argument that would lead us to expect a different result. If, as is the case in the numerical example, an individual's indifference curves are convex, then as the park moves closer to his home, he will be willing to pay less for further moves of the same distance. The person who is getting farther from the park will be willing to pay progressively more to reverse these movements that are unfavorable to him. It would seem that when the park is next to one man's house, the distant individual would often be willing to pay most heavily to have the park moved toward himself. As we shall see, this argument is valid; the apparent contradiction

14. A utility function $U = f(x,y)$ will have convex indifference curves if $f_{11}f_2^2 - 2f_{12}f_1f_2 + f_{22}f_1^2$ is less than 0. An individual will take fair actuarial gambles on one of the arguments if either f_{11} or f_{22} is positive. Thus, if f_{11} is positive, the individual will have convex indifference curves as well as be a risk preferrer on the first argument of the utility function if $f_{11}f_2^2 + f_{22}f_1^2 < 2f_{12}f_1f_2$. If the complementarity between the goods, as measured by f_{12}, is great enough, an individual can be a risk taker on both arguments but still have convex indifference curves.

with the preceding argument is not real.

The Paradox Resolved. A numerical example will help make matters clear. Consider a situation in which $r_A + r_B = \$15,000$. Figure 5 displays the utility-possibility frontier for this example. The portion of the frontier between q and x represents outcomes in which the park is between 0 and 1. Quite obviously, lotteries on outcomes q and x can be employed to dominate all points on this portion of the utility-possibility frontier for certainty outcomes. The dashed line gives the expected utilities for such lotteries.

Figure 5



Values Along the Utility Possibility Frontier with Certainty Outcomes

Characteristics of the Point	p Max of U_A	q End Pt. of Tangent Line	s Inflection Pt.–Park Just at 0	t	u	v	w Inflection Pt.–Park Just at 1	x End Pt. of Tangent Line	y Max of $\frac{U_A + U_B}{2}$	z Max of U_B
U_A	118.39	117.02	110.37	80	50	20	-1.96	-3.46	-3.47	-4.43
U_B	-9.56	-6.10	-2.55	11.22	32.66	66.62	115.18	118.90	118.92	119.31
$\$r_A$	15,000	14,584	12,704	10,486	7,951	4,799	1,139	119	114	0
$\$r_B$	0	416	2,296	4,514	7,049	10,201	13,861	14,881	14,986	15,000
m	0	0	0	.193	.410	.682	1	1	1	1

A look at the terminal resource distribution and the location of the park for points q and x is instructive. Neither point represents what we would think of as being a relatively equal distribution of resources or utility. This may not matter to us if neither of these two points is preassigned and if the outcome is the result of a mutually-agreed-upon lottery. The difficulty with the maximize-expected-utility approach is that in general it will not require that a lottery be conducted on two such unequal points. Rather, it will choose with certainty one point that will likely be highly unequal.[15] In the numerical example, it chooses point y, which places the park at 1 and has $r_A = \$114$.

The explanation of the apparent paradox is now evident. If individuals are risk preferrers with respect to gambles on one variable, but have convex indifference curves, Pareto-optimal schemes will always require that the man who "gets a lot" of one valued variable will get a lot of the other. It thus turns out that the man who has all of the variable for which there is risk preference will be so wealthy on the other variable that he will be unwilling to sell any of the first variable at the price that his universally poorer coinhabitant would be willing to pay.

Further Apparent Paradoxes in a Many Variable World. Both individuals may be risk-averse with respect to gambles on either variable alone. This would imply that every one-variable lottery would be dominated by some certainty outcome. But with two or more variables involved in the lotteries, the certainty outcomes may be dominated.

Consider the utility function

$$U = r^{.8}(1 - d)^{.8}$$

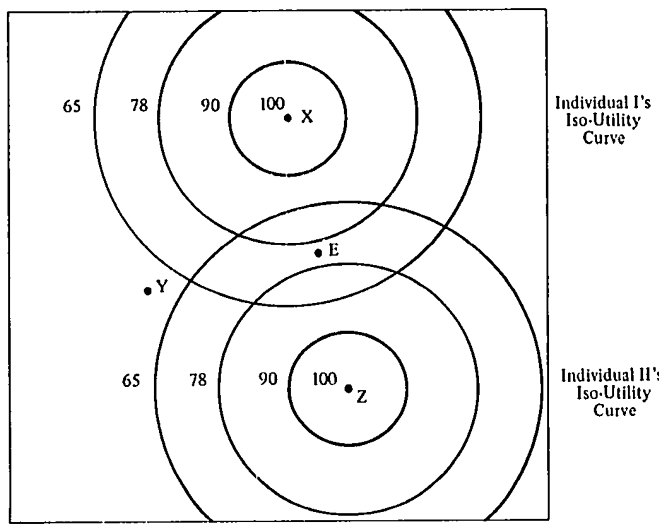
where d represents distance from the park. If both individuals had this utility function (which displays risk aversion on either valued variable considered separately), the Pareto-optimal outcomes would all involve lotteries on the extreme points, the points that place one individual next to the park and give him all of the joint resources. This example has a direct analogy to production theory. Two production processes can each offer increasing returns to scale, but diminishing marginal returns to either factor. For any set of prices, the value of output is maximized by assigning all factors to one or the other production process.

With many qualitative issues, the intriguing question arises as to the number of underlying variables that are really represented. The answer leads to the question, what is meant by risk aversion when a qualitative dimension is under consideration? Consider a situation where two individuals have strictly conflicting ordinal preferences over a set of alterna-

15. In the situation in which the slope of the dashed lottery line is exactly -1, the maximize-expected-utility criterion will make society indifferent among all the points along the dashed line. Otherwise, the tangency of the utility possibility frontier and a line of slope -1, a line that gives a constant sum for $U_A + U_B$, will be in either the upper-left or lower-right section of the utility possibility frontier.

tives, the first choice of one is the last for the other, etc. The alternatives' qualities are described by vectors. If no lotteries ever dominate compromise certainty choices, we might conclude that at least for this range of alternatives the individuals are risk-averse. Conversely, if each in-between certainty alternative is dominated by a lottery, we might conclude that for some scaling at least both individuals are risk preferrers. But the second of these conclusions may not be justified, as Figure 6 demonstrates.

Figure 6



Both individuals have strictly convex preferences, and their utility functions are concave. Yet on the choice among X, Y and Z, they would both prefer a 50-50 lottery on X and Z to Y for sure. The reason that these two risk-averse individuals jointly prefer a lottery to a certainty outcome is that points such as E, efficient compromise points, are not available. (In the town-park example, the viable alternative Y would represent a location 3/8 of a mile perpendicular from the center of town.) If only a limited range of outcomes is under consideration, individuals may jointly prefer lotteries to certainty outcomes, despite the fact that they are risk-averse.[16]

The Implications of Lotteries. For three distinct reasons individuals may prefer lotteries on alternatives to certainty outcomes. (1) For certain scalings of the underlying spectrum, both may be risk preferrers. (2) Though both are risk-averse for any one-variable lotteries, they may become risk preferrers once two-variable lotteries are under consideration. (3) The alternatives may be restricted in such a way to rule out the efficient com-

16. This observation has some important implications for social choice procedures that rarely allow lotteries to compete as alternatives. See Zeckhauser [15] and Fishburn [5].

promise certainty alternatives that would dominate a lottery.

The upshot of all this in real life may be that Adams and Brown agree to a lottery on two outcomes, one of which makes Adams well-off on both variables and the other of which makes Brown well-off on both variables. That is fine and does not appear to be inequitable. They both have a chance to come out the winner, and both would prefer the lottery rather than accept some in-between outcome for certain.

We might feel a bit less comfortable if we followed the maximize-expected-utility criterion from the outset. Then an unequal outcome would be uniquely determined, and in some sense one man would have had no chance. Perhaps this would be a good place to employ a maximin criterion, at least with respect to expected utility for either individual. We would draw a 45° line out through the Pareto frontier and conduct the lottery that was represented by the intersection of the two curves.

This example throws up some difficult questions of equity that often arise in the presence of uncertainty. Should expected utilities be employed in calculations of social welfare? The inability to identify in advance the particular individuals who will benefit from the application of the maximize-the-sum-of-individuals'-utilities criterion makes it a form of implicit lottery: does this justify the fairness of its use? Questions of this nature take on new importance in the context of the next Section.

IV. PREFERENCES FOR PARTICULAR TYPES OF SOCIETAL OUTCOMES

Individuals may have preferences for societal outcomes that go beyond their concern for values of variables that enter directly into their own consumption or for the way in which the outcomes are chosen. Altruism may play a role; thus other individuals' utilities may matter. One way to treat this possibility is to consider a total utility function for each individual that combines two classes of arguments: those that enter directly into his own consumption and those that are derived from the utilities of others. For expositional purposes, we call an individual's total utility function his aggregate function.[17]

Surely most individuals would be willing to sacrifice some of their own consumption bundle if this would enable the consumption bundle of others to be increased significantly. If the sacrificer were rich and the

17. Individuals may not be indifferent to the ways in which other individuals derive their utilities. This would be the case if those who make transfers to others like to indicate how they should be spent. If this were the case, the structure of the aggregate functions would be more complicated.

Aaron and von Furstenberg [1], for example, observe that housing expenditures by the poor may provide greater external utility benefits than other forms of expenditure. When the welfare of both recipients and transferrers is considered, a transfer in kind of housing may be more efficient than cash grants. The recipients alone may prefer such a transfer in kind. They may perceive that given the greater external benefits of a transfer in kind, the magnitude of such a transfer may be substantially greater than cash grants from the same donors.

gainers poor, this willingness would likely be reinforced. Any such willingness would imply that the individual's aggregate function was not identical with his utility function, that it also contained the utilities of others as arguments. We might expect that aggregate functions would display some bias toward egalitarian outcomes. In their analysis of "Pareto Optimal Income Transfers" [6], Hochman and Rodgers used such a bias to explain why a rich man may find his utility improved when he makes transfers to his poor fellow citizen.

A less extreme thought experiment can reveal the presence of this type of utility structures. Consider a world in which there were two types of consumption bundles: large and small. All transfers are ruled out. Ask Adams and Brown which of two lotteries they would prefer. The candidates are Lottery I (.5, Adams small, Brown large; .5, Adams large, Brown small), and Lottery II (.5,. Adams small, Brown small; .5, Adams large, Brown large). If Adams and Brown would both prefer Lottery II, the "if either-gets-rich-we-both-get-rich" lottery, they have some egalitarian preferences. If this preference is strong, they might continue to prefer Lottery II even if the probability of the both-small outcome went to .6.

Given such egalitarian preferences, or any other preferences that lead to a divergence between individuals' aggregate functions and their utility functions, a utility-possibility graph will be inadequate to its usual tasks. What we want is a graph that gives the constrained maximum frontier for individuals' welfares as indicated by their own preferences. In this case it is the constrained frontier for values of the aggregate functions. Fortunately, so long as aggregate functions have as arguments only the utilities of individuals in the society, the desired frontier can be calculated directly from the utility-possibility frontier[18] (not from the expected utility possibility frontier).

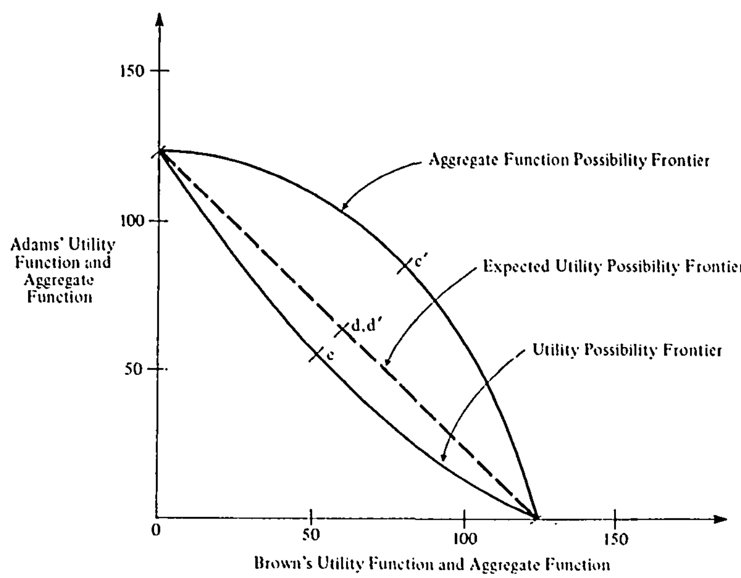
If individuals have strong egalitarian biases, the value-of-the-aggregate-function possibility frontier may be concave when the utility-possibility frontier (for certainty outcomes) is not. Thus, if the outcome-choosing criterion is to maximize the expected value of a randomly chosen individual's aggregate function, we may end up ruling out lotteries (on what are undoubtedly unequal outcomes) even though lottery outcomes may lie on the expected-utility-possibility frontier.

Consider an example where the two individuals' aggregate functions include a term that pushes toward equality. The aggregate functions are for Adams $U_A + U_A \cdot U_B$, and for Brown $U_B + U_B \cdot U_A$. Figure 7 shows the

18. The problem becomes more complicated if values of individuals' aggregate functions rather than of their utility functions enter the aggregate functions of others. On this subject see Zeckhauser and Fels [19].

way in which a convex utility-possibility frontier given by the function $(300 - U_A)^2 + (300 - U_B)^2 = 12{,}065$ is mapped into the value of the aggregate-function-possibility frontier. Point *c*, which provides an equal distribution of utility, is transformed to point *c′* on the aggregate function possibility frontier. Point *d*, which is achieved by conducting a lottery on the extreme points *e* and *f*, offers a higher expected utility than *c* to each individual. But it produces highly unequal after-lottery outcomes. With the posited aggregate functions, point *d* produces point *d′* (with which it happens to coincide) in the aggregate function possibility space. Observe that *c′* dominates *d′*.

Figure 7



Lotteries on unequal outcomes may be considered unattractive if individuals have altruistic-type preferences of an egalitarian nature. All the arguments that were made above with respect to selecting a point on the utility-possibility frontier can also be made with respect to choosing a point on the value-of-the-aggregate-function possibility frontier.

Should the chosen societal outcome be based on the values of aggregate functions or on the values of utility functions? Should it be chosen by a maximin criterion, by a maximize-expected-value criterion, or by some completely different criterion? The importance of these questions ranges well beyond the need for answers in the economic models considered here. They are important because they relate to basic philosophical issues with regard to social orderings. In our search for the elusive social welfare function, we might wish to employ the function that would have been accepted in the formulation of an original social contract. The men agreeing to such a contract might be unable to identify their positions in future society. In this way they might act like two rational individuals who did not know whether they would end up as Adams or Brown in our model. The criterion they would adopt to choose an outcome would depend upon the basic structure of preferences for outcomes and for methods for choosing outcomes. Given present knowledge, thoughts on these matters must be somewhat speculative. The possible payoffs are great; such speculation would seem worthwhile.

VI. SUMMARY AND CONCLUSION

Complexities arise when the traditional public goods model is applied to aspects of public goods other than quantity. Individuals may differ in their ordinal preference rankings for qualitative characteristics and thereby rule out the use of more-less scalings for the attribute. If a more-less scheme is not available, marginal analysis may lead us away from the optimum optimorum. That marginal analysis may lead us astray is not surprising, since we are dealing with significant deviations from traditional public goods models, many important in their own right. We have no underlying spectrum such that for all individuals more is better. Furthermore, where qualitative characteristics of a good are concerned (as opposed to quantitative amounts of alternative goods), there can be no assurance that each consumer will prefer "means to extremes." This implies we can not hope to find neat tangency solutions.

None of these new wrinkles gets ironed out even if it is assumed, as in this paper, that budgetary amounts have already been determined. Although this paper seeks alternative procedures for settling individuals' conflicting preferences for the qualitative characteristics of a particular public good, the discussion relating choice mechanisms to underlying principles is perfectly general and could equally well be applied in other areas.

Getting to the Pareto frontier involves the maximization of a weighted sum of individuals' utilities. The classic unsolved problem of welfare economics is to find a way to assign the weights. Consideration of individuals' von Neumann-Morgenstern utility functions may provide some assistance in this task. If utility functions are normalized with regard to some common reference points, the required weights are derived implicitly, if as may seem ethically appealing, individuals' normalized marginal utilities are treated symmetrically. This procedure provides a way to include distributional considerations in decisions on the provision of public goods. But this approach does not relate to initial conditions (the *status quo*) and thus overlooks bargaining considerations. Altered standards for normalization will be required if the selected outcome is in any way to reflect asymmetrical positions at the start.

In a very different vein, the maximization of the sum of utilities may lead to a very unequal *ex post* result. Where individuals are risk preferrers with regard to one argument of their utility functions, this unhappy situation will be greatly aggravated. The situation will be aggravated in a different way if individuals' preferences relate to other individuals' utilities as well as to their own.

There are other criteria that might be used to make social choices. One possible candidate is the criterion to maximize the minimum utility of any individual. (If the utility-possibility frontier is concave, its application will lead to complete equality of utilities.) If the criteria by which social choices are made are of importance to individuals, then the maximin criterion should be promoted from possible candidate to significant contender. While this paper does not attempt to make interpersonal comparisons of utility, it does suggest one standard that can be applied to simulate the process of making them and another that can be employed to circumvent the need for making them. These standards are highly imperfect, first efforts. They are meant merely to be suggestive, but they do indicate the types of raw inputs that are needed to make social choices.

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